The OEIS, Mathematical Discovery, and Insomnia

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The OEIS Foundation and Rutgers University

ACMES Conference, May 2016

Outline

- Introduction
- Coincidences
- Low-hanging fruit from new sequences
  - Strange recurrences
  - Sequences from number theory
- Music and videos

Introduction

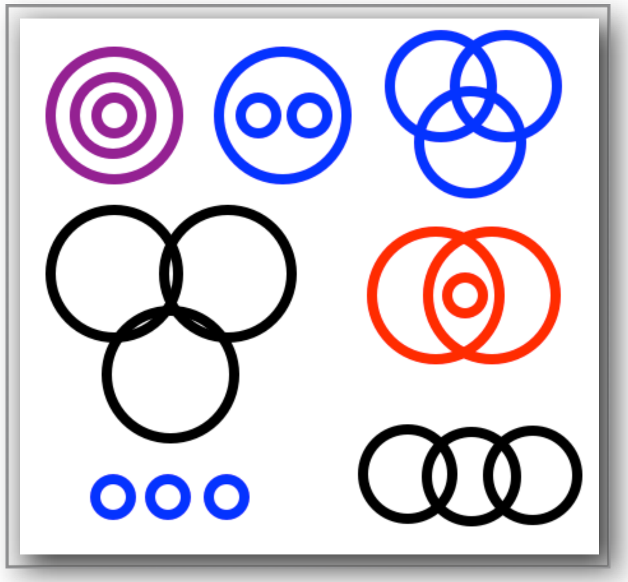
oeis.org

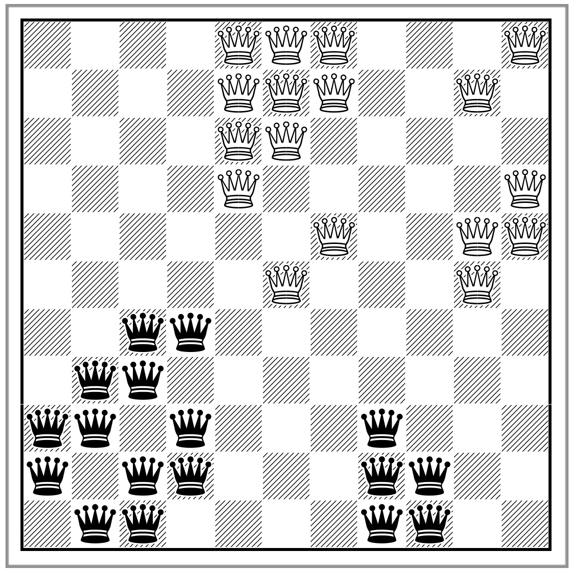
Facts about the OEIS

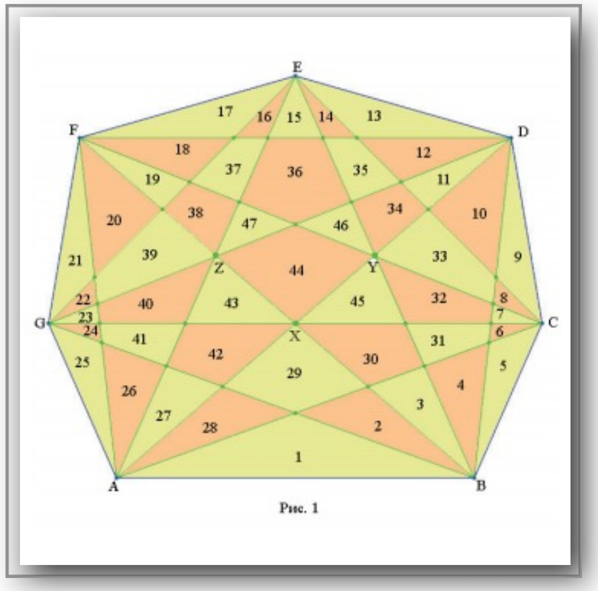
- Accurate information about 272000 sequences
- Definition, formulas, references, links, programs
- View as list, table, graph, music!
- 50 new entries, 50 updates every day
- Traffic: 155 GB/month, 9 million hits/month
- 5000 articles and books cite the OEIS
- Often called one of best math sites on the Web
- Maintained by NJAS and a dedicated group of unpaid editors. More editors are badly needed.

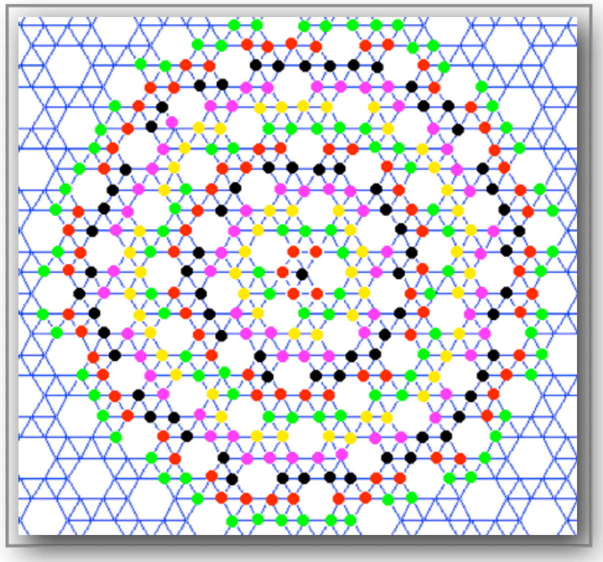
The new poster, on the OEIS Foundation web site, http://oeisf.org

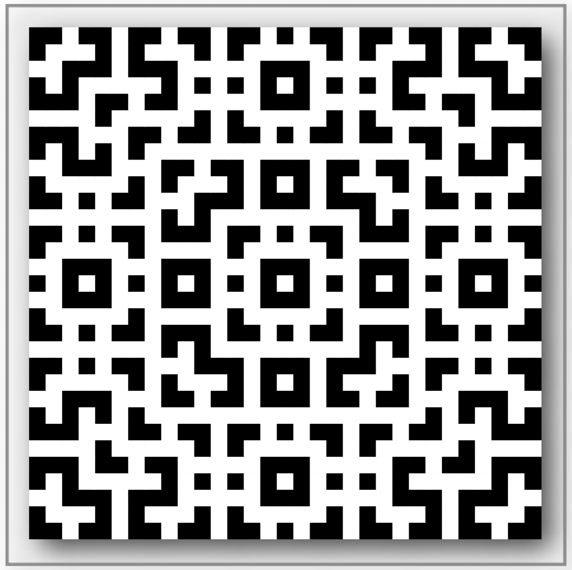
OEIS.org

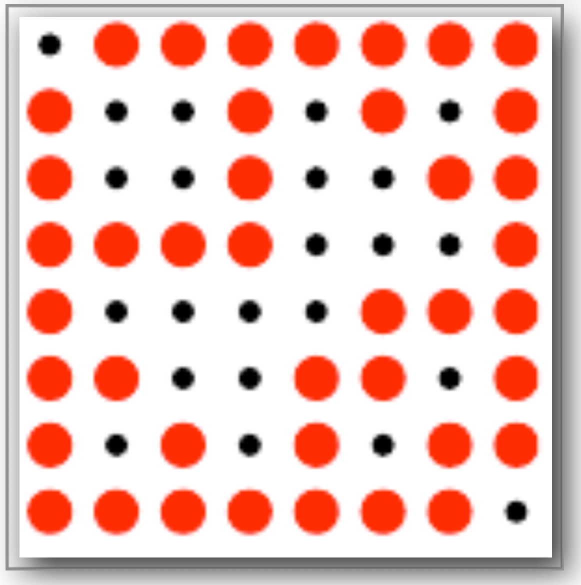


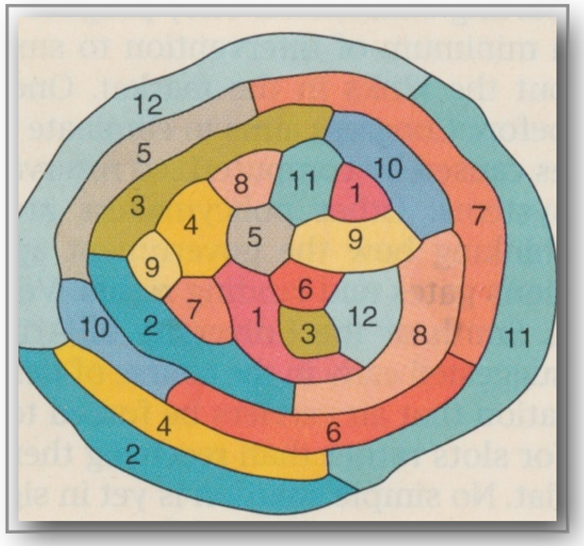


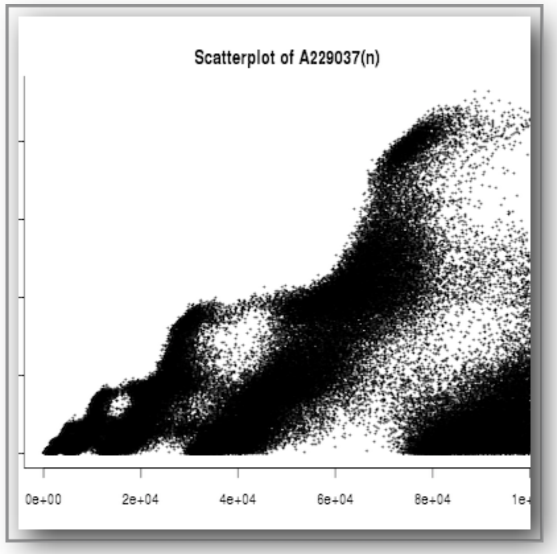


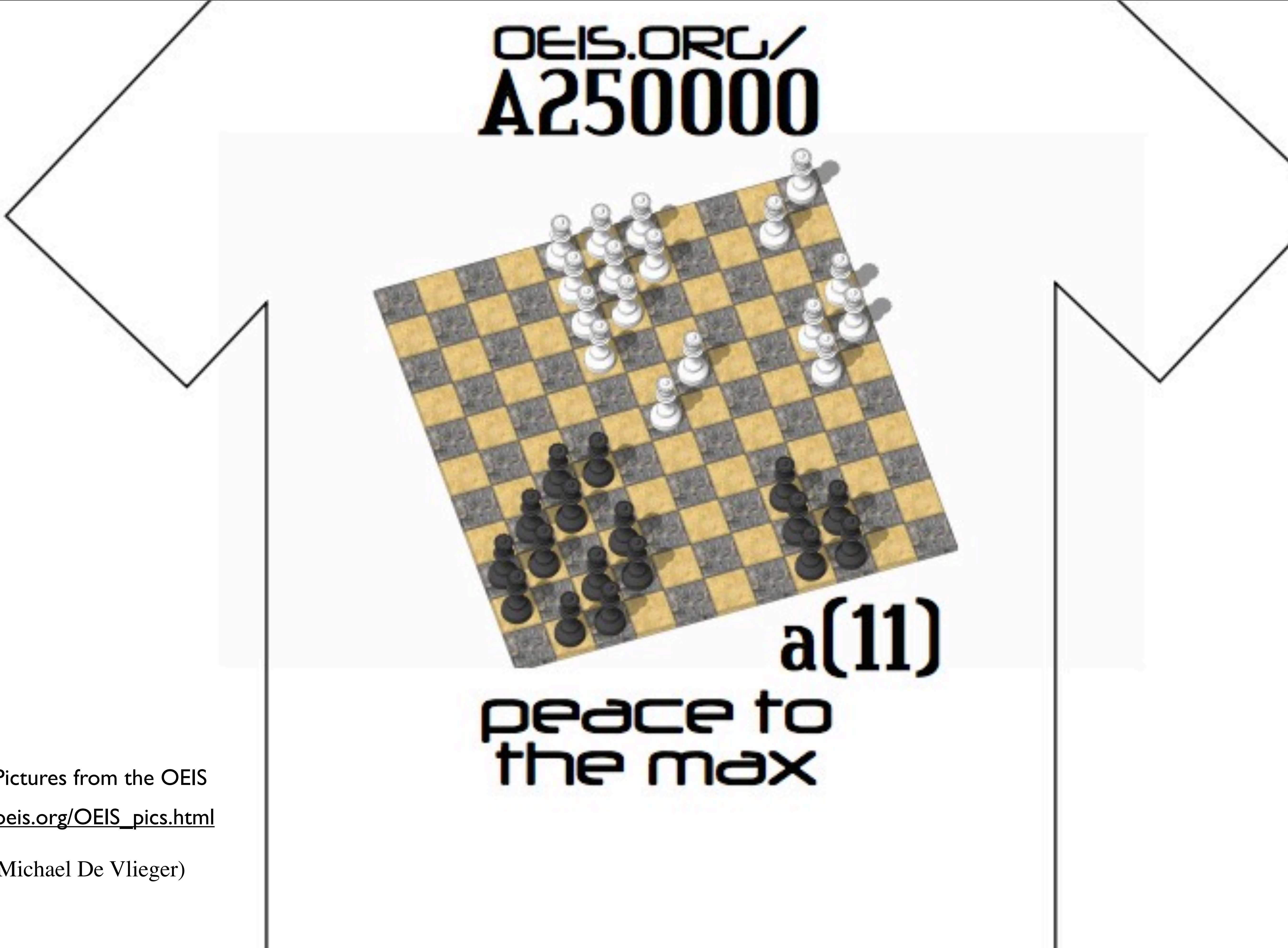












Pictures from the OEIS

oeis.org/OEIS_pics.html

(Michael De Vlieger)

Coincidences

- Nolan Wallach, Variety of commuting matrices
- A. S. Fraenkel, From enmity to amity
- P. Aluffi, Degrees of projections of rank loci
- Ping Sun, Enumeration of standard Young tableaux of shifted strips with constant width
- `Sandpiles and Dominoes`

Nolan Wallach, A029729, 1999

Degree of the variety of commuting nXn matrices

1, 3, 31, 1145 (10 workstations running for 5 hours, 1993)

Also, ratio of vector elements of the ground state in the loop representation of the braid-monoid Hamiltonian H = Sum_i (3 - 2 e_i - b_i) with size 2n and periodic boundary conditions. (B. Nienhuis and J. de Gier, 2005)

Martins, Nienhuis, Rietman, An intersecting loop model as a solvable super spin chain, Phys. Rev. Lett. 1998.
di Francesco, Zinn-Justin, Inhomogeneous model of crossing loops and multidegrees of some algebraic varieties, 2004.
Razumov, Stroganov, Combinatorial nature of ground state vector of O(1) loop model, Theor. Math. Phys. 2004.

1, 3, 31, 1145, 154881, 77899563, 147226330175, 1053765855157617, 28736455088578690945, 3000127124463666294963283, 1203831304687539089648950490463

Aviezri Fraenkel
From Enmity to Amity, Am. Math. Monthly, 2010

Sloane's influential ***On-Line Encyclopedia of Integer Sequences*** is an indispensable research tool in the service of the mathematical community. The sequence A001611 listing the "Fibonacci numbers + 1" contains a very large number of references and links. The sequence A000071 for the "Fibonacci numbers − 1" contains an even larger number. Strangely, resentment seems to prevail between the two sequences; they do not acknowledge each other's existence, ... Using an elegant result of Kimberling, we prove a theorem that links the two sequences amicably. We relate the theorem to a result about iterations of the floor function, which introduces a new game.

P. Aluffi, Degrees of projections of rank loci, arXiv:1408.1702

"After compiling the results of many explicit computations, we noticed that many of the numbers $d_{n,r,S}$ appear in the existing literature in contexts far removed from the enumerative geometry of rank conditions; we owe this surprising (to us) observation to perusal of [Slo14]."

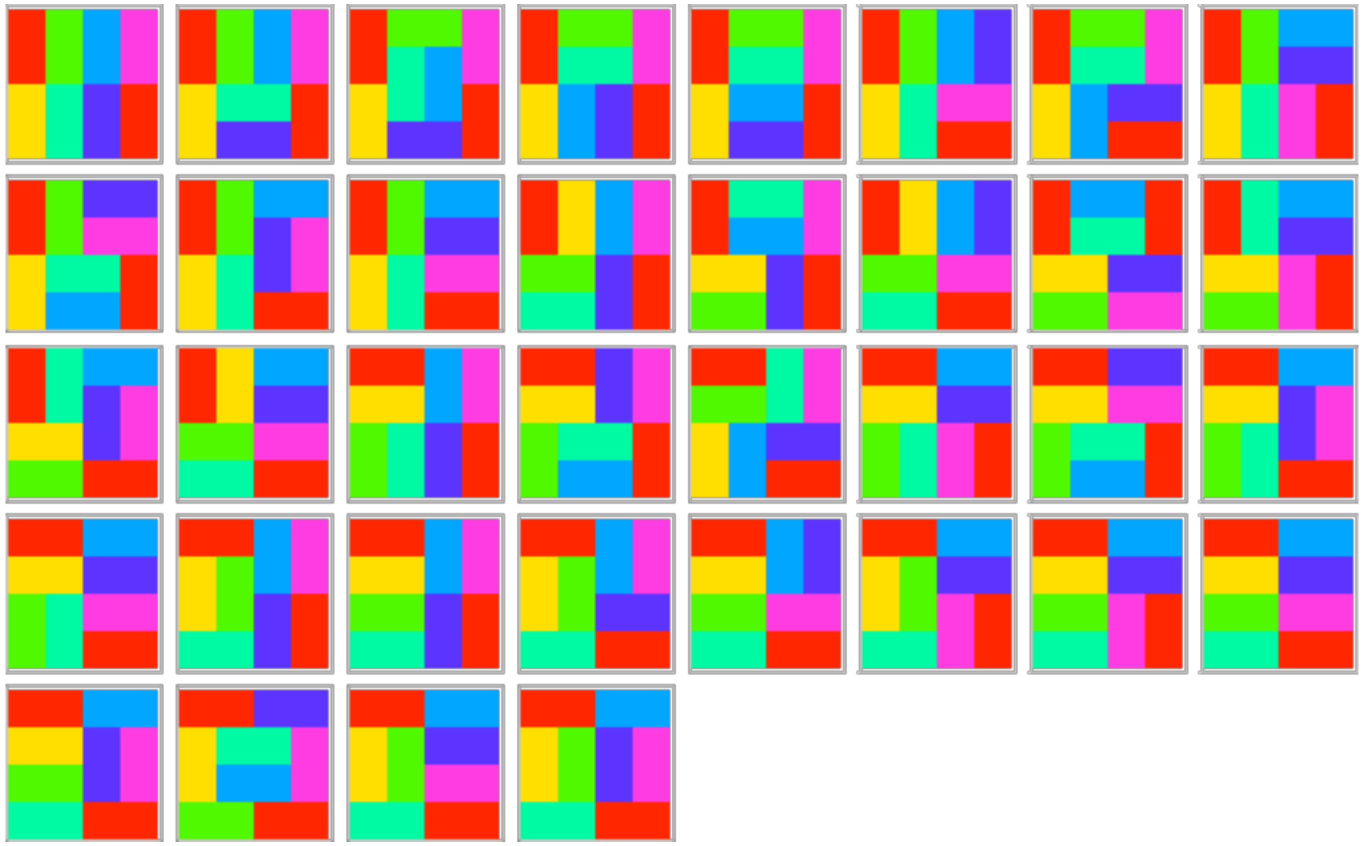
Ping Sun (Shenyang, China), Enumeration of standard Young tableaux of shifted strips with constant width, arXiv 2015, finds same sequences as enumerated by R. H. Hardin when counting n X k matrices containing a permutation of 1, ...,nk in increasing order rowwise, columnwise, diagonally and (downwards) antidiagonally, with empirical recurrences.

From Tim Gowers's Blog, May 10 2016

It has long been a conviction of mine that the effort-reducing forces we have seen so far are just the beginning. One w[illegible]d more fully is in the creation of amazing new databases, something I once asked a Mathoverflow question about. I recently had cause (while working on a research project with a student of mine, Jason Long) to use Sloane's database in a serious way. That is, a sequence of numbers came out of some calculations we did, we found it in the OEIS, that gave us a formula, and we could prove that the formula was right. The great thing about the OEIS was that it solved an NP-ish problem for us: once the formula was given to us, it wasn't that hard to prove that it was correct for our sequence, but finding it in the first place would have been extremely hard without the OEIS.

Tiling a Square with Dominoes

36 ways to tile a 4X4 square



a(2)=36

1, 2, 36, 6728, 12988816, 258584046368, 5306047752196000, ...

(A4003)

$$a(n) = \prod_{j=1}^{n} \prod_{k=1}^{n} \left(4\cos^2 \frac{j\pi}{2n+1} + 4\cos^2 \frac{k\pi}{2n+1} \right)$$

(Kastelyn, 1961)

Last year:

Laura Florescu, Daniela Morar, David Perkinson, Nicholas Salter, Tianyuan Xu, Sandpiles and Dominoes, 2015

1, 2, 36, 6728, 12988816, 258584046368, 5306047752196000/5, ... !!

(A256043)

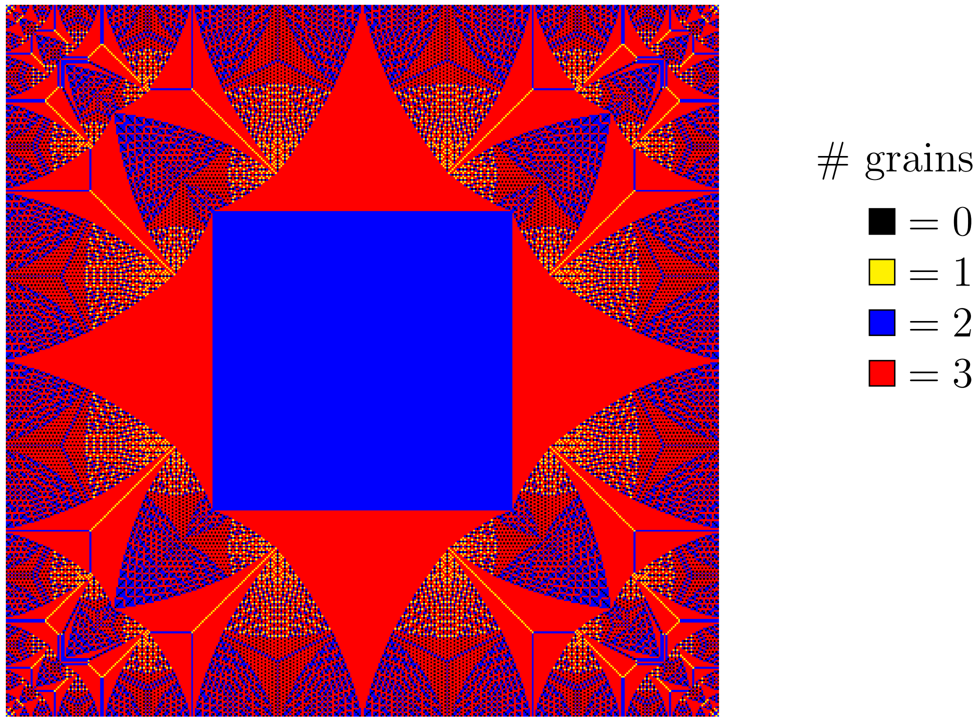


Figure 1: Identity element for the sandpile group of the 400 × 400 sandpile grid graph.

Two Sequences That Agree For a Long Time

$$\left\lfloor \frac{2n}{\log 2} \right\rfloor$$ = A078608

$$\left\lceil \frac{2}{2^{1/n} - 1} \right\rceil$$

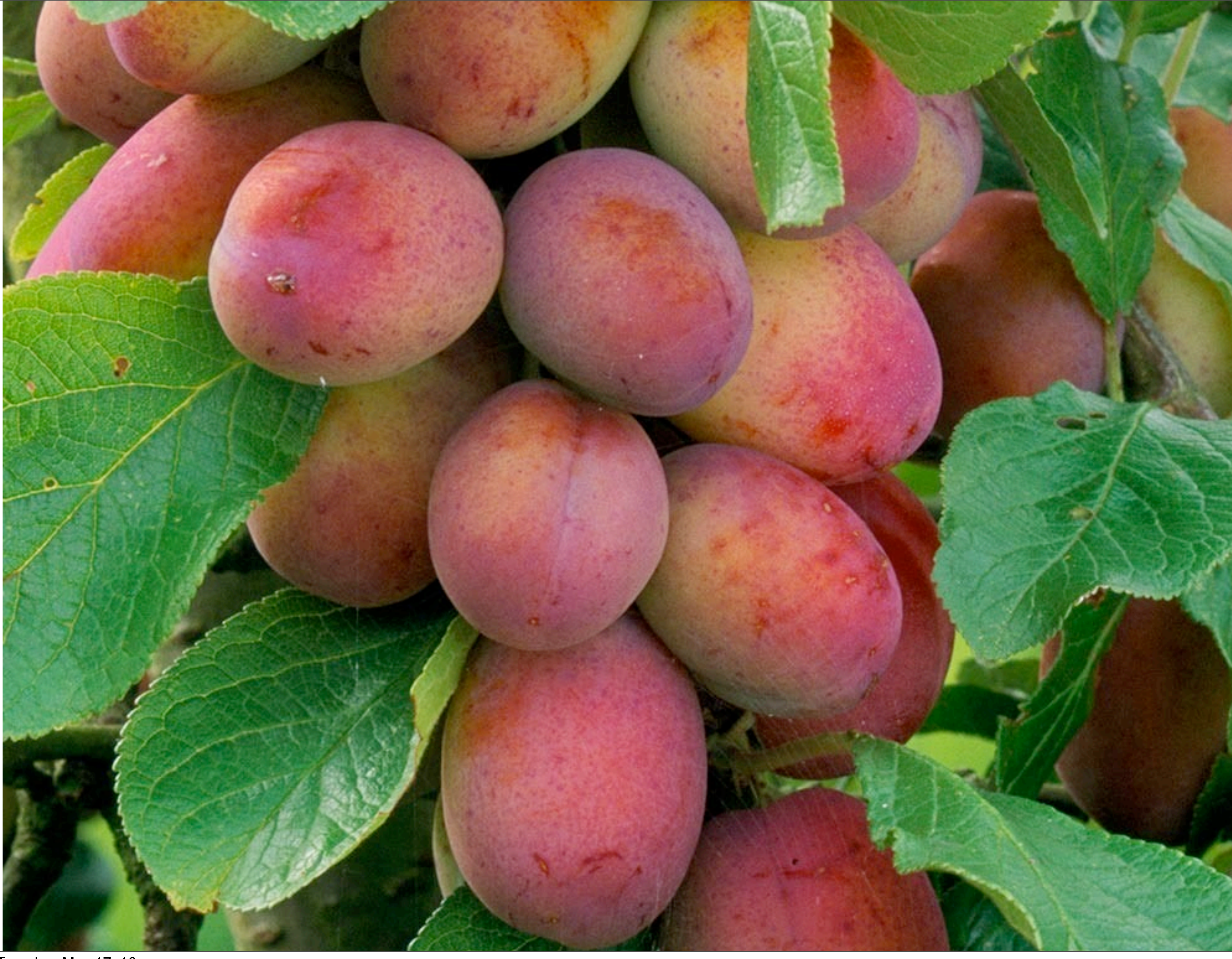
Differs for first time at n =

777451915729368

(see A129935)

Low-Hanging Fruit from the OEIS

Some new problems for the ghosts of Fermat, Gauss, Euler, ...



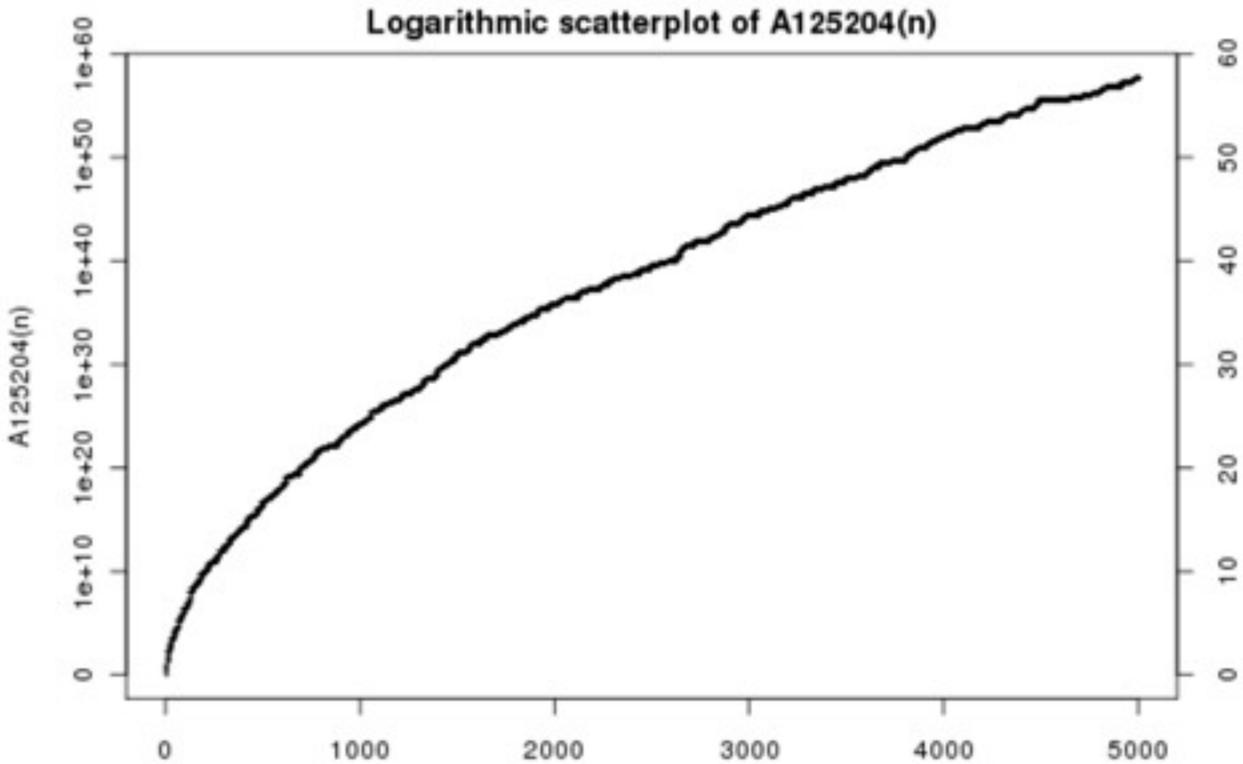
Strange Recurrences

- Modified Fibonacci
- Reed Kelley
- A recurrence that looks ahead
- Van Eck's sequence

Modified Fibonacci

a(n) = a(n-1) + a(a(n-1) mod n) with a(0)=0, a(1)=1

A125204, Leroy Quet, 2007



Log plot of 5000 terms

Explain!

Similar to A268176, January 2016, also not analyzed

Reed Kelley's Sequence A214551

14th century Narayana cows sequence A930:

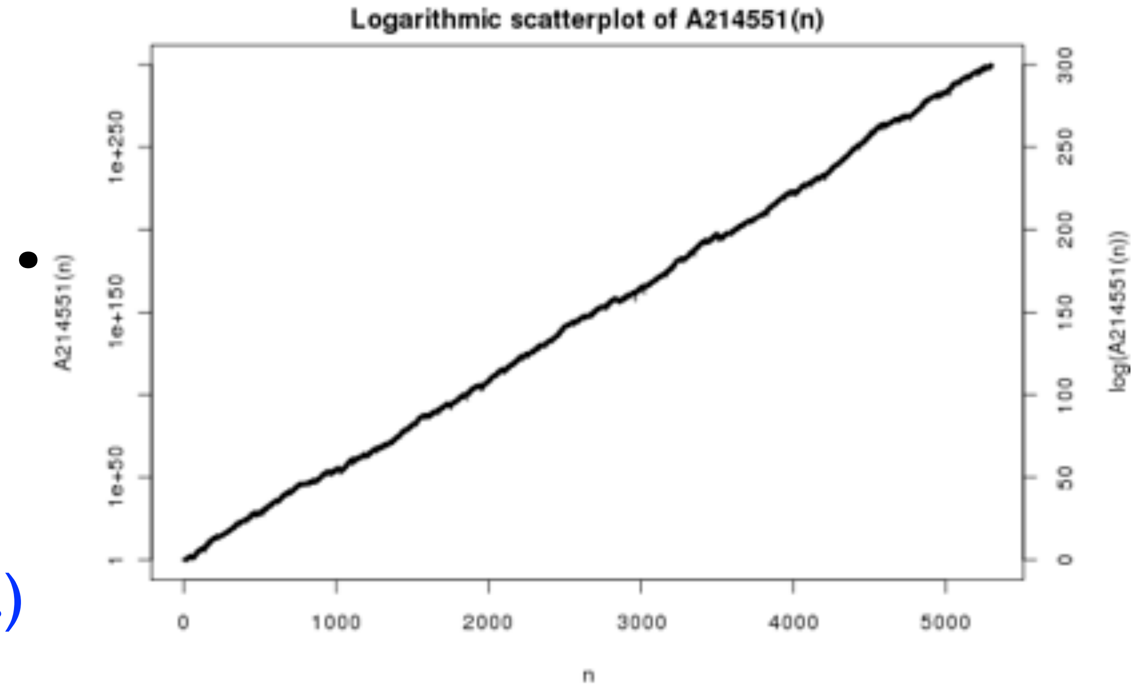
$$a(n) = a(n-1) + a(n-3)$$

1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, ...

Reed Kelley, 2012:

$$a(n) = \frac{a(n-1) + a(n-3)}{\gcd\{a(n-1), a(n-3)\}}$$

```
1, 1, 1, 2, 3, 4, 3, 2,
3, 2, 2, 5, 7, 9, 14, 3,...
```



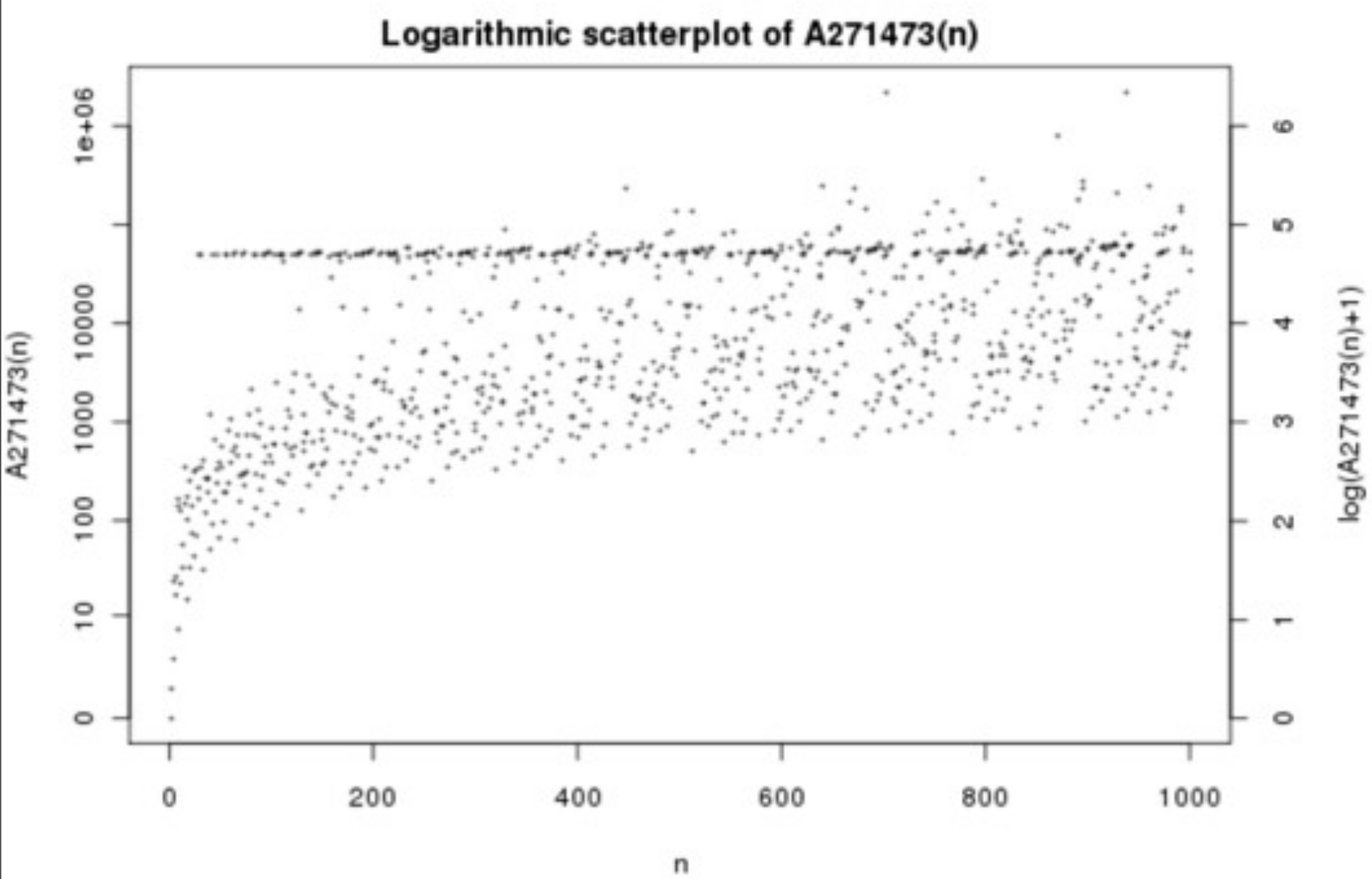
(Have guesses, but nothing is proved.)

A recurrence that looks ahead

a(2k) = k+a(k), a(2k+1) = k+a(6k+4) with a(1)=0.

A271473, suggested by 3x+1 sequence A6370
and new A266569

Apr 8 2016



Explain!

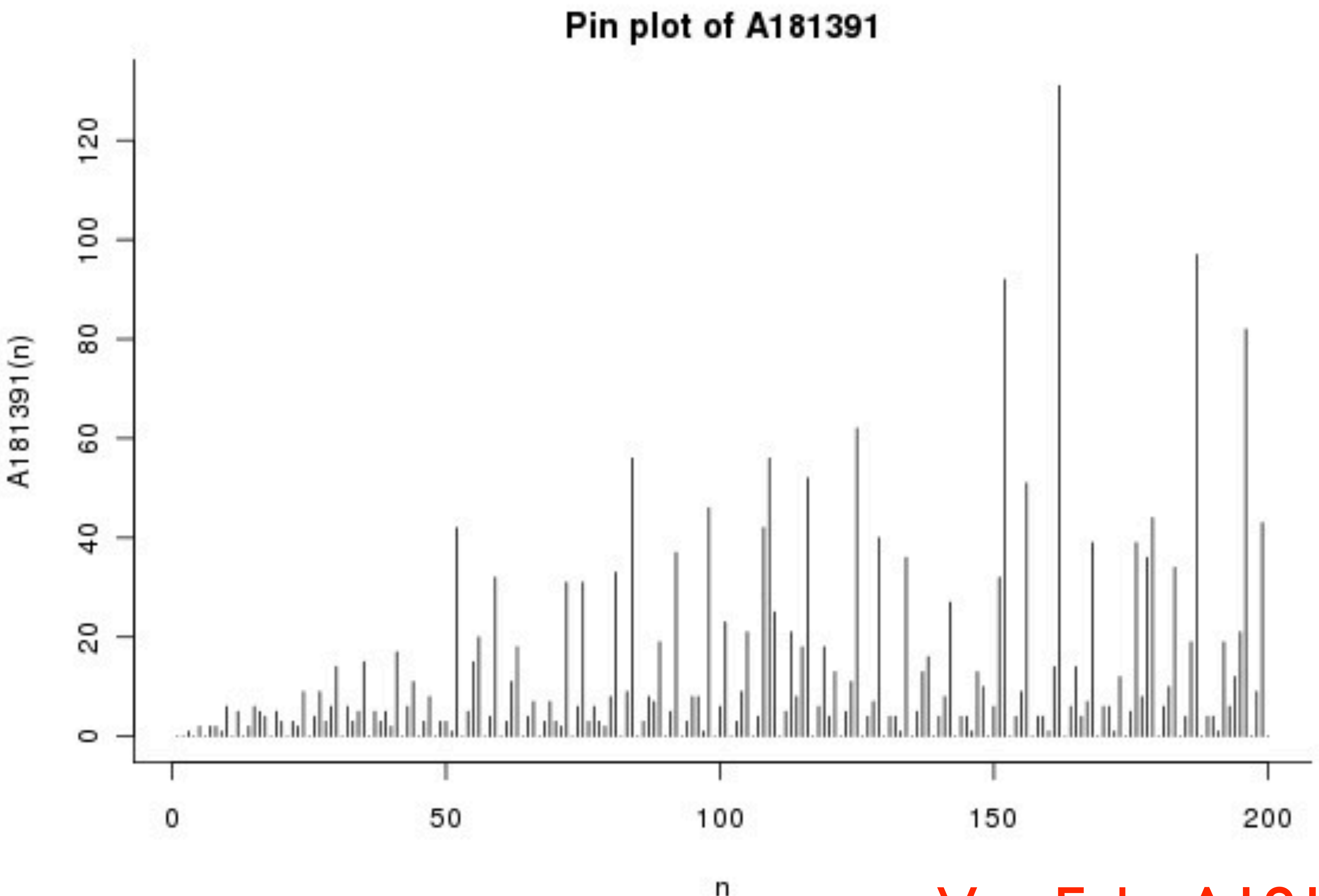
Jan Ritsema van Eck's Sequence

```
0, 0, 1, 0, 2, 0, 2, 2, 1, 6, 0, 5,
0, 2, 6, 5, 4, 0, 5, 3, 0, 3, 2, 9,
0, 4, 9, 3, 6, 14, 0, 6, 3, 5, 15, 0,
5, 3, 5, 2, 17, 0, 6, 11, 0, 3, 8, 0, ...
```

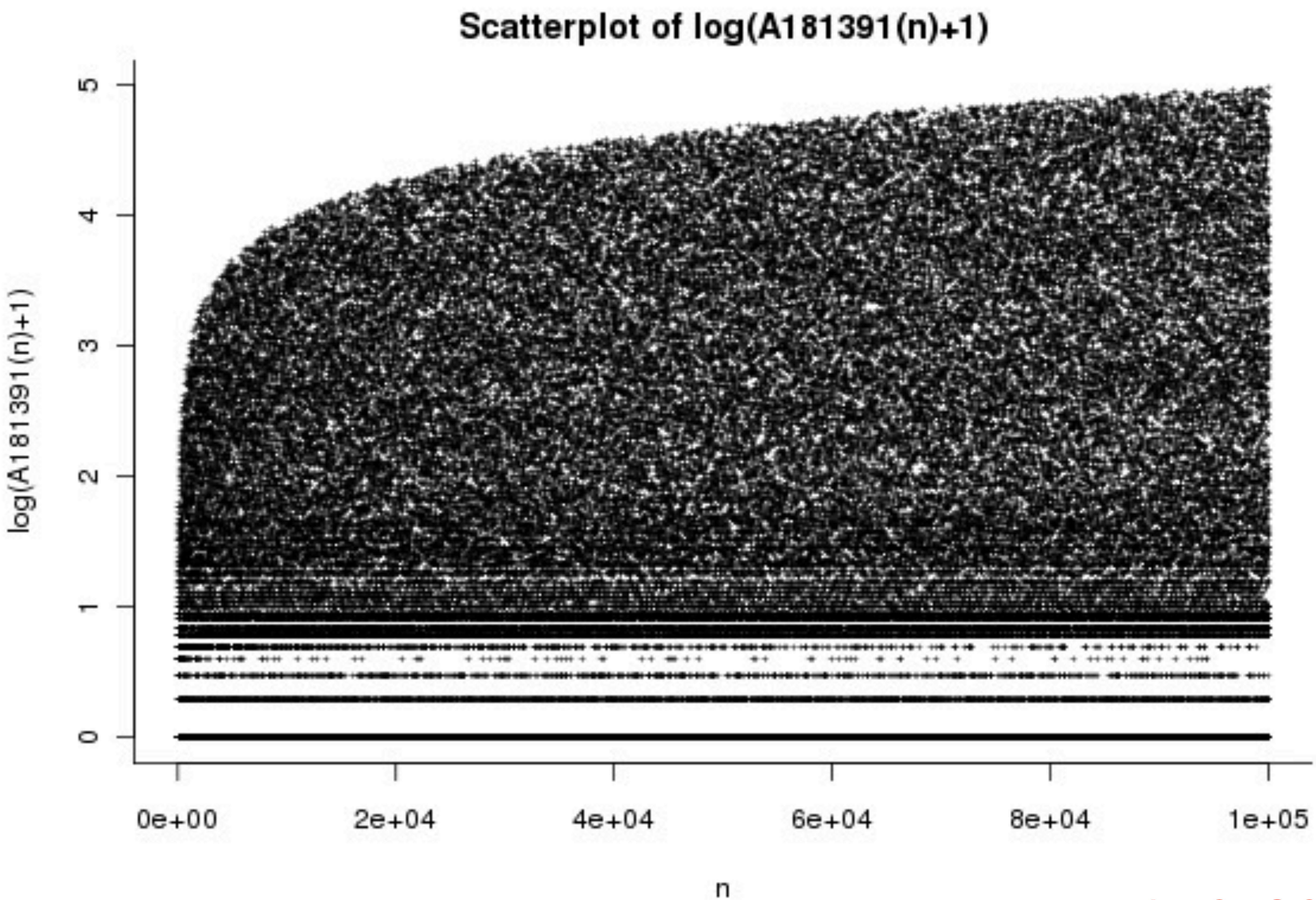
a(n): how far back did we last see a(n-1)?
or 0 if a(n-1) never appeared before.

Van Eck: A181391

A181391 as a graph:



Van Eck: A181391



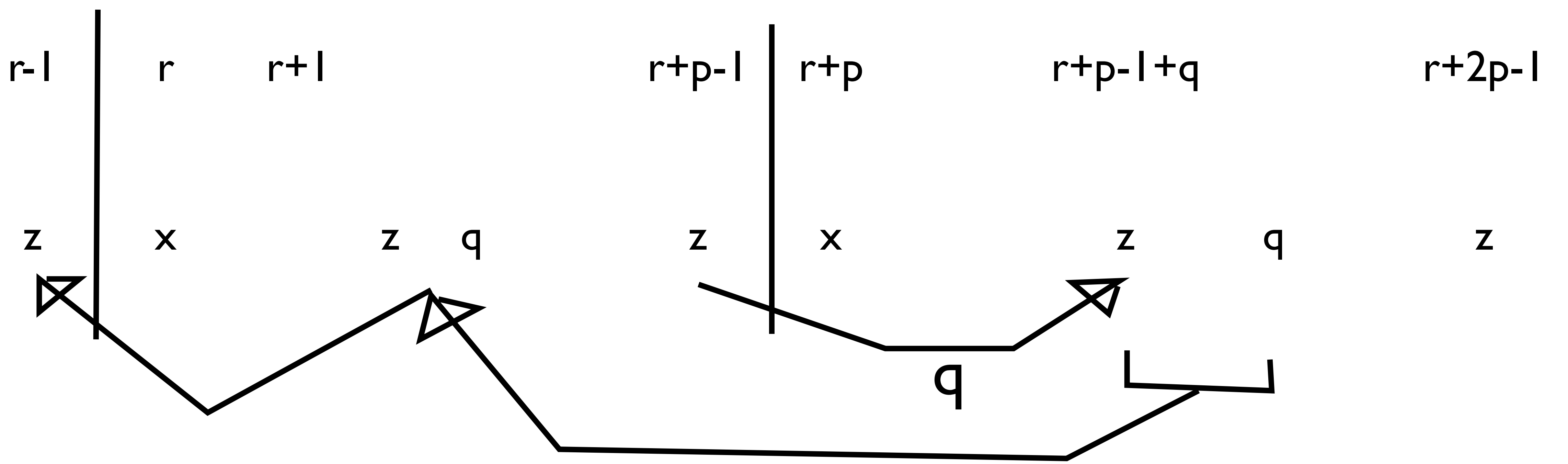
Van Eck: A181391

Thm. (Van Eck) There are infinitely many zeros.

Proof: (i) If not, no new terms, so bounded.
Let M = max term.
Any block of length M determines the sequence.
Only M^M blocks of length M.
So a block repeats.
So sequence becomes periodic.
Period contains no 0's.

Van Eck: A181391

Proof (ii). Suppose period has length p and starts at term r.



Therefore period really began at term r − 1.

....

Therefore period began at start of sequence.
But first term was 0, contradiction.

Van Eck: A181391

It seems that:

lim sup a(n) / n = 1

Gaps between 0's roughly log_10 n

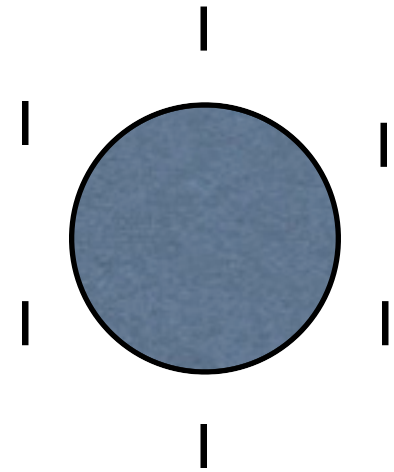
Every number eventually appears

Proofs are lacking!

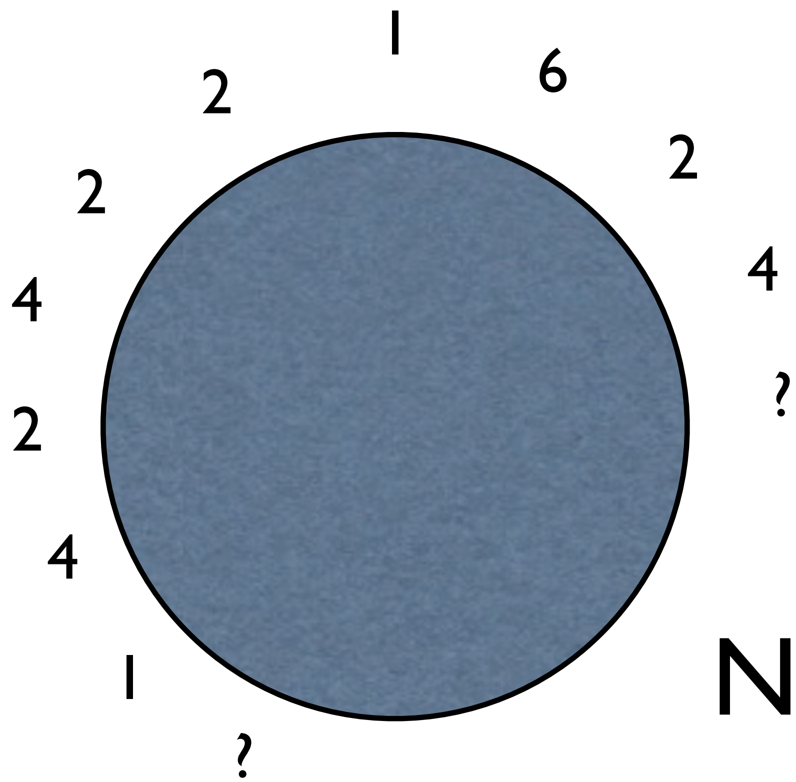
Van Eck: A181391

Van Eck: A181391

Conjecture:
There is no
nontrivial cycle



Trivial cycle



Nontrivial cycle ?

(David Applegate: Only trivial cycles of length up through 14)

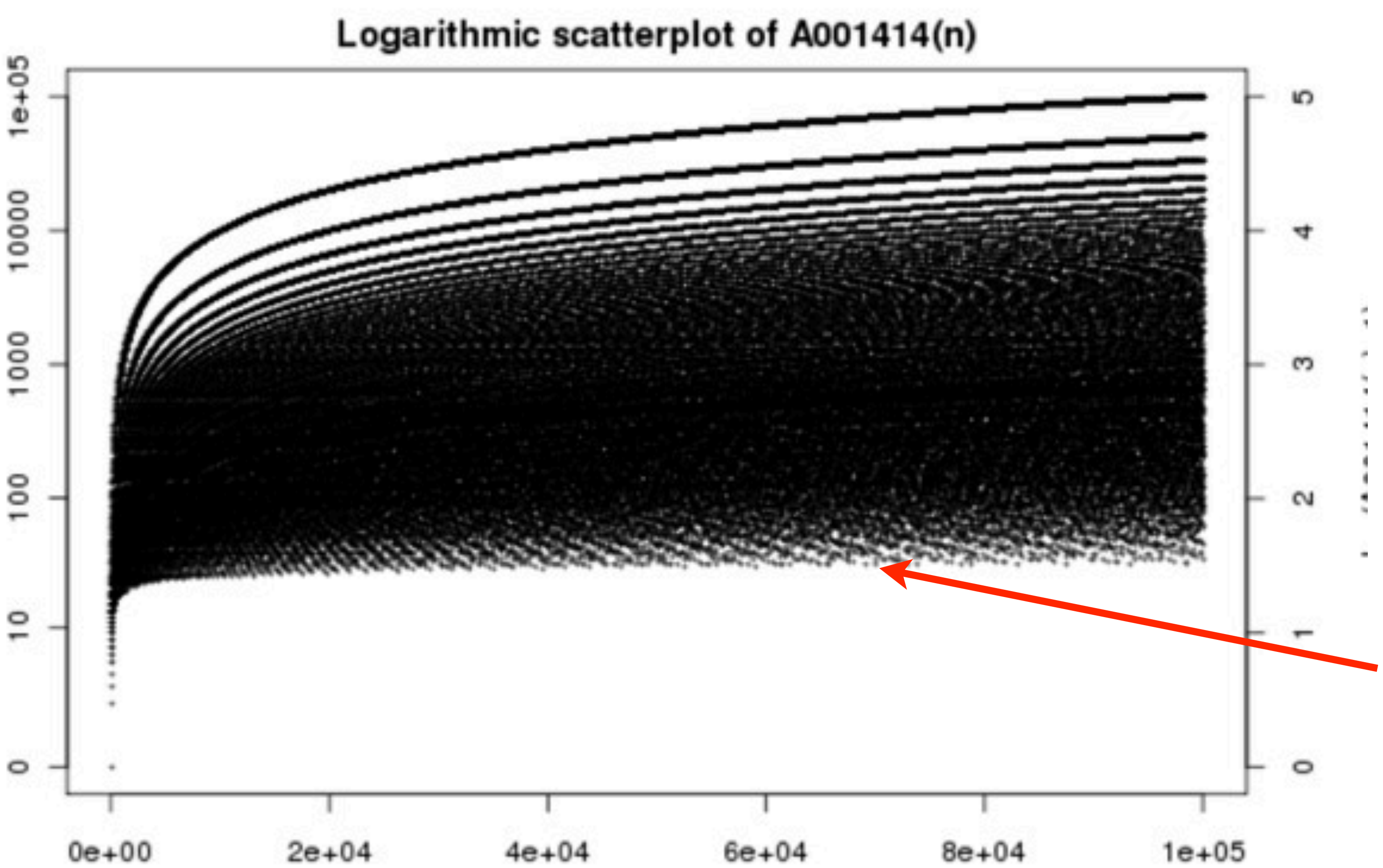
Number Theory

- Sum of primes in sum of previous terms
- Yosemite graph?
- Leroy Quet's prime-producing sequence
- 999999000000
- A memorable prime
- When is 12345...n a prime?
- The Fouriest transform

New sequence related to potency of n

"Potency" of n = "Integer log" of n = sum of primes dividing n (with repetition) MacMahon 1923

0, 2, 3, 4, 5, 5, 7, 6, 6, 7, 11, 7, 13, ...



A001414

Explain these stripes? - Allan Wechsler

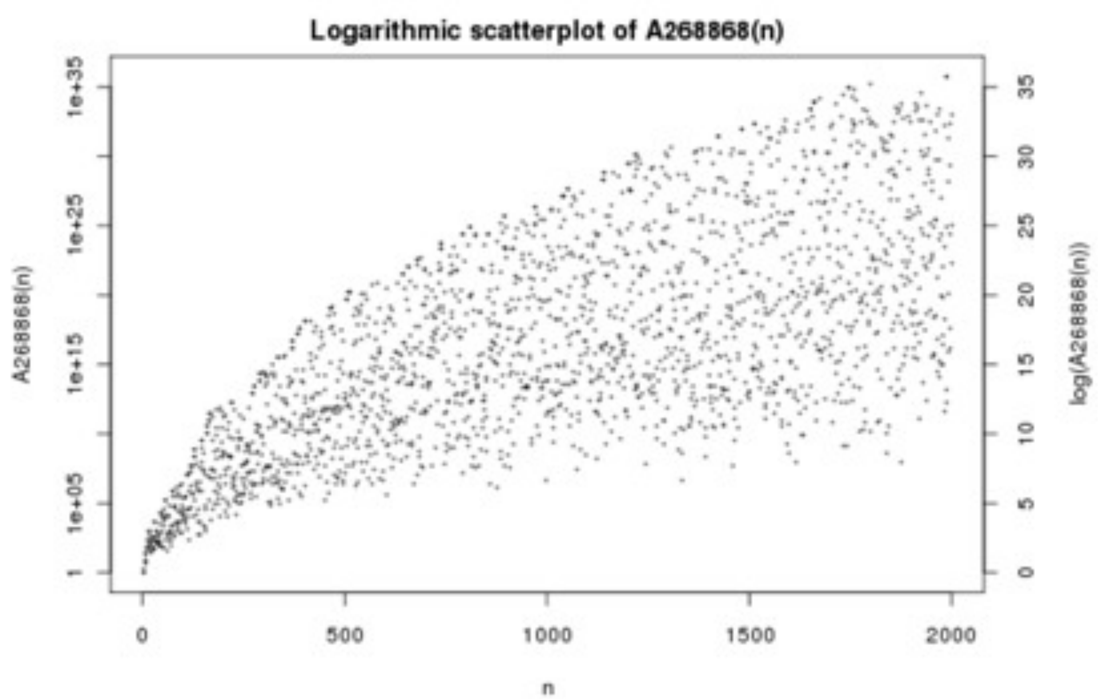
a(n) = sum of prime factors of sum of all previous terms

(with repetition, starting 1, 1)

1, 1, 2, 4, 6, 9, 23, 25, 71, 73, 48, 263, 265, 120, 911, 913, 552, 192, 85, 27, 35, 53, 296, 66, 455, 289, 48, 188, 5021, 5023, 159, 190, 379, 946, 900, 600, 97, 204, 118, 512, 87, 148, 3886, 23291, 23293, 71, 896, 11812, 60, 41359,

1+1+2+4+6 = 14 = 2 x 7 gives 2+7 = 9

A268868, David Sycamore, Feb 2016



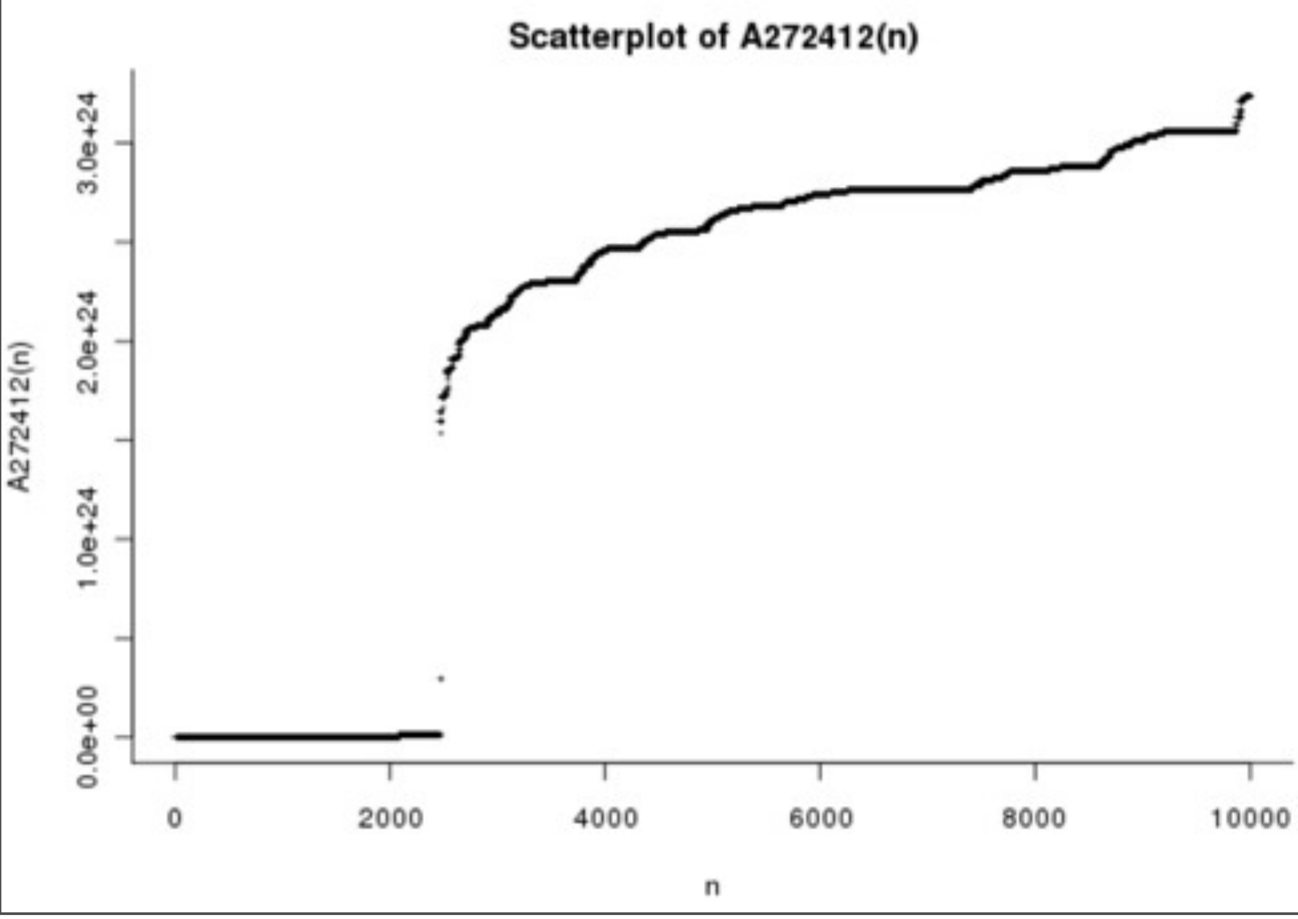
Explain!

Generalize!

Yosemite Graph?? (A272412)

Numbers n such that sum of divisors (A203(n)) is a Fibonacci number (in A45)

Random combination of 2 sequences, except look at the graph:



Altug Alkan, Apr 29 2016

Have 10000 terms but need a lot more

Hostadter’s Q-sequence
A5185

Leroy Quet’s Prime-generating sequence
A134204

Franklin Adams-Watters
A166133

“About your cat, Mr. Schrödinger—I have good news and bad news.”

(The New Yorker, March 2015)

A134204

Leroy Quet's Prime-Producing Sequence

n

0	1	2	3	4	5	6	7	8	9	10
2	3	5	7	13	17	19	23	41	31	29

p q

q = smallest missing prime such that n divides p + q

10 divides 31 + 29

$p + q = kn$

$q = -p + kn$

Dirichlet: OK unless p divides n

Does the sequence exist?

800 000 000 terms exist

999999000000

Max Alekseyev, A261206, Aug 11 2015

If $\lceil n^{1/k} \rceil \mid n$ for all k then n ≤ 999999000000 (conj.)

1, 2, 4, 6, 12, 36, 132, 144, 156, 900, 3600, 4032, 7140, 18360, 44100, 46440, 4062240, 9147600, 999999000000

No more terms below 10^16

999999000000 (cont.)

Th. 1

$$\lceil \sqrt{n} \rceil \mid n \iff n = \left\lfloor \frac{M}{2} \right\rfloor \left\lceil \frac{M}{2} \right\rceil \quad \text{for some } M$$

(the quarter-squares, A002620)

Pf.

$$\lceil \sqrt{n} \rceil = m + 1 \Leftrightarrow m^2 + 1 \leq n \leq (m+1)^2$$

$$\text{Say } n = m^2 + 1 + i$$

$$\text{So } i = m - 1 \text{ or } 2m, \quad n = m(m+1) \text{ or } (m+1)^2$$

$$M = 2m + 1 \text{ or } 2m + 2$$

Example:

$$999999000000 = \left\lfloor \frac{1999999}{2} \right\rfloor \left\lceil \frac{1999999}{2} \right\rceil$$

999999000000 (cont.)

Th. 2

$$\lceil n^{1/3} \rceil \ | \ n \ \Leftrightarrow \ n = m^3 + 1 + \lambda(m+1), \ 0 \le \lambda \le 3m$$

for some m (A261011)

Example: With $m = 9999, \lambda = 29897$,

$$m^3 + 1 + \lambda(m+1) = 999999000000$$

If both Th 1 and Th 2 apply, get A261417 :

1, 2, 4, 6, 9, 12, 36, 56, 64, 90, 100, 110, 132, 144, 156, 210, 400, 576, 702, 729, 870, ...

And so on ?

A Memorable Prime

12345678910987654321

When is 123...n-1 n n-1...321 prime?

It is a square: $11...1^2$ for $n \leq 9$.

Prime for n=10, 2446 (Shyam Gupta, PRP only), ...

NOT IN OEIS!

Or, in base b, when is 123...b-1 b b-1...321 prime?

Prime for b =

2, 3, 4, 6, 9, 10, 16, 40, 104, 8840 (PRP)

(David Broadhurst, Aug 2015, A260343)

When is 12345...n a prime?

Concatenate 1 through n in base 10.
When is the first prime?

n=19 fails:

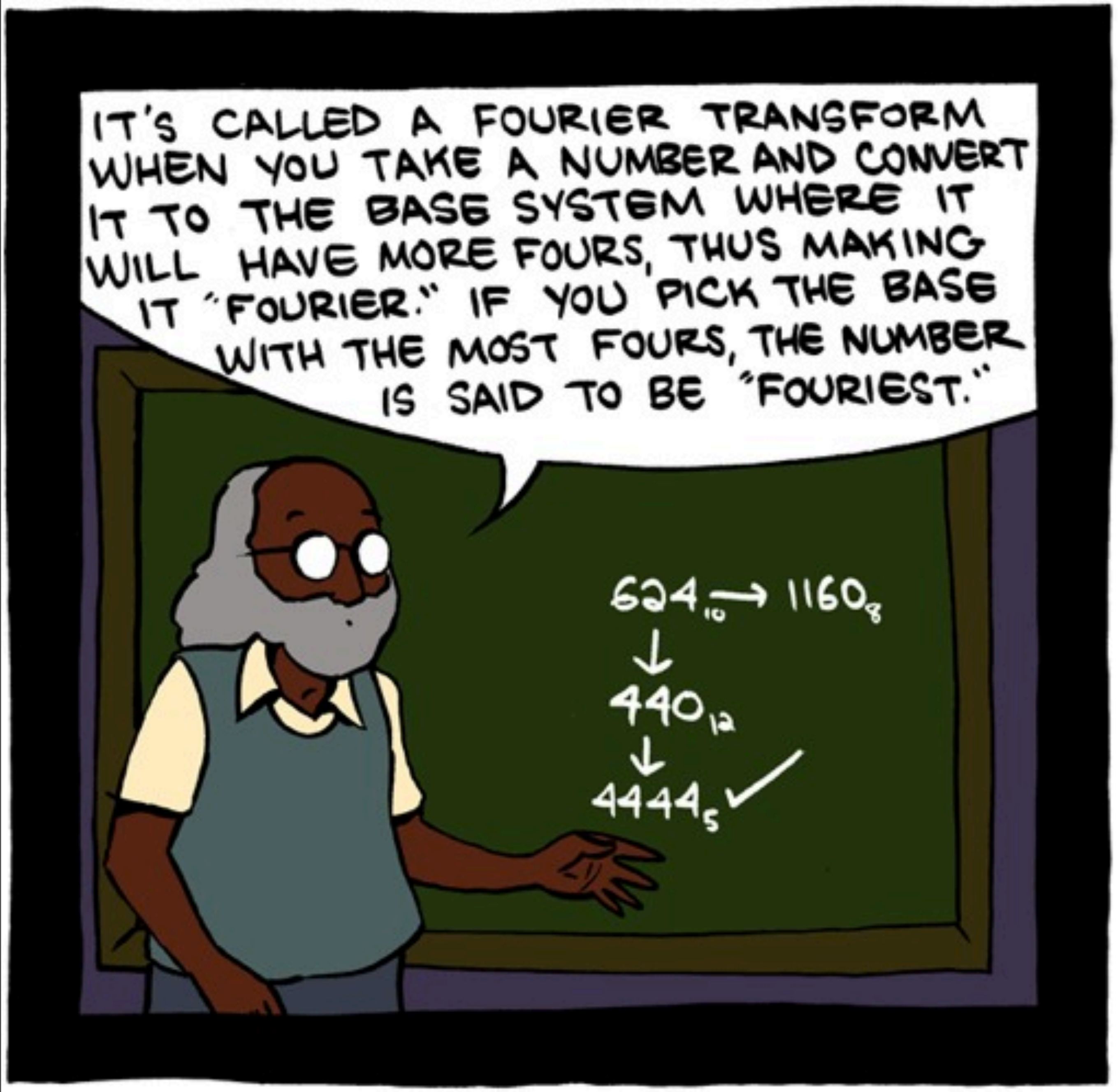
12345678910111213141516171819
= (13) (43) (79) (281) (1193) (833929457045867563)

All n < 344870 fail.

Sequence is surely infinite.

See A7908 for progress of the search

The Fouriest Transform of n



Teaching math was way more fun after tenure.

Zach Weinersmith, Saturday Morning Breakfast Cereal

Write n in that base b >= 4 where you get the most 4's

a(10)=14 (use base 6)

A268236

0, 1, 2, 3, 4, 11, 12, 13, 20, 14, 14, 14, 14, 14, 24, 14, 24,..

“Music” and Videos

Reminder: New keywords “hear” and “look”

Pascal’s triangle

A7318

Hofstadter Q sequence
A5185

$a(1) = a(2) = 1$; $a(n) = a(n-a(n-1)) + a(n-a(n-2))$ for $n > 2$.

wt(n) and 4^wt(n) together

(A120 and A102376, Taiko drum and xylophone)

Martin Paech's arrangement of A242353

Recaman's sequence   A5132

(Midi "instrument" FX-7)

Samuel Vriezen, Toccata III (2001)

Faure, Prelude, Op. 103, #3

(in G Minor)

Videos about sequences

Charles McKeague, <u>Fibonacci numbers</u>

Dale Gerdemann, <u>Fibonacci tree</u>

Christobal Vila, <u>Nature by numbers</u>

Robert Walker, <u>Golden Rhythmicon</u>

Gordon Hamilton, <u>Wrecker ball sequence</u> (Recaman's sequence)

There are nearly 200 videos, movies, animations in the OEIS - we need more!

The OEIS needs more editors!

Lovely new problems every day

https://oeis.org/draft has the queue

Contact njasloane@gmail.com if interested