Design of Asymmetric Multiple Description Lattice Vector Quantizers

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Abstract

We consider the design of asymmetric multiple description lattice quantizers that cover the entire spectrum of the distortion profile, ranging from symmetric or balanced to successively refinable. We present a solution to a labeling problem, which is an important part of the construction, along with a general design procedure. This procedure is illustrated using a $\mathbb{Z}^2$ lattice. We also evaluate its rate-distortion performance and compare it to known information theoretic bounds.

I Introduction

A multiple description source coder generates a set of binary streams or descriptions of a source sequence, each with its own rate constraint. The transmission medium may deliver some or all of the descriptions to the decoder. The objective is to minimize the distortion between the source sequence and the decoded sequence when all the descriptions are available, while ensuring that the distortion obtained by decoding only a subset of the descriptions remains below a pre-specified value that depends on the subset. With D descriptions, we associate a vector of length 2^D, each element of which is a distortion constraint for a given description subset. This distortion vector is referred to as the distortion profile.

In recent years, multiple description coders have received considerable attention, driven by the interest in packet voice and video communications. Most of the work has centered around the successively refinable and balanced cases (with the exception of [1]), which are in a sense two extreme cases of the distortion profile. Successive refinement coders find application in networks with a priority structure whereas balanced codes are useful in networks that do not have such a structure, the best example being the Internet.

In this paper we propose a structured scheme that bridges the two schemes, in the sense that it allows for an arbitrary distortion profile (within reason). By making the descriptions have different distortions, one can show that the quantizer behavior can range from balanced (where each description is equally important) to a strict hierarchy (where the loss of some descriptions could make the decoding impossible). The design is proposed for a lattice vector quantizer, but the general principle of unbalanced multiple description coding can be extended to a host of other quantizers. This could include trellis coded quantizers, unstructured vector quantizers etc. This could potentially allow us to incorporate channel (or network route) reliability information into the transmission. Also, it might be a useful way to allow for less intrinsic wastage of network traffic as any/all descriptions could be used for source decoding without necessarily waiting for the more important descriptions to be available (as in successive refinement).

For previous work on the information theoretic aspects of the multiple description problem see [2, 3, 4, 5, 6]. The problem of designing quantizers for the multiple description problem has been considered in [7, 1, 8, 9, 10, 11]. The work presented here is for the asymmetric case, and extends the work in [9] which considered the balanced case. Unlike the work in [1], we do not use a training approach; instead we use the geometry of the underlying lattice to solve a labeling problem. Other approaches to multiple description coding based on overcomplete expansions are presented in [12, 13, 14] and methods based on optimizing transforms and predictors are presented in [15, 16, 17].

The paper is organized as follows. The source coding problem is formulated in Section II, the design method is described in Section III, numerical results are presented in Section V.

II Preliminaries

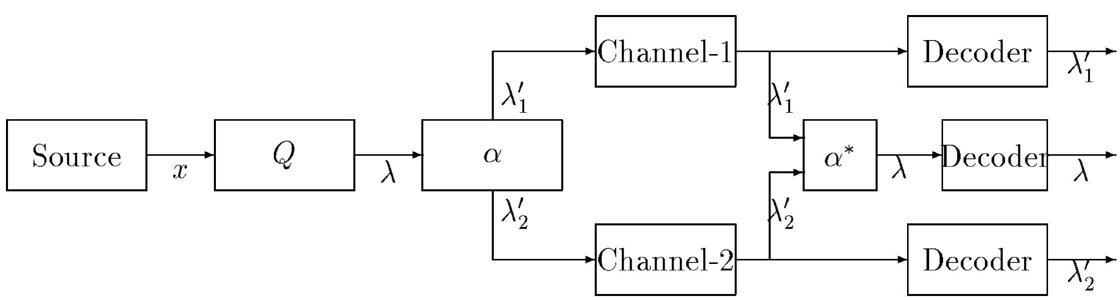

Figure 1: Block diagram of a multiple description vector quantizer.

A block diagram of a multiple description vector quantizer (MDVQ) with a lattice codebook is shown in Fig. 1. An L-dimensional source vector x is first encoded to the nearest vector λ in a lattice $\Lambda \subset \mathbb{R}^L$. We will write $\lambda = Q(x)$. Information about the selected codevector λ is then sent across the two channels, subject to rate constraints imposed by the individual channels. This is done through a labeling function α. At the decoder, if only channel 1 works, the received information is used to pick a vector λ'_1 from the channel 1 codebook. If only channel 2 works, the information received over channel 2 is used to pick a codevector λ_2' from the channel 2 codebook. If both channels work, it is assumed that enough information is available in order to recover λ.

We will assume that the channel 1 and channel 2 codebooks, denoted Λ_1 and Λ_2, respectively, are sublattices of Λ. The index $[\Lambda_i : \Lambda]$ is denoted N_i, $i = 1, 2$. In addition we will also assume that Λ_i is geometrically similar to Λ, i.e., Λ_i can be obtained by scaling and rotating Λ. Additionally, we will also require a sublattice Λ_s of $\Lambda_1 \bigcap \Lambda_2$, which is chosen such that it is also geometrically similar to Λ and its index $N_s = [\Lambda_s : \Lambda]$, satisfies $N_s = N_1 N_2$. For lower numerical computation in the design we choose a sublattice Λ_{lcm} of $\Lambda_1 \bigcap \Lambda_2$ which has index $N_{lcm} = \mathrm{LCM}(N_1, N_2)$.

Since the information sent over channel 1 is used to identify a codevector $\lambda_1 \in \Lambda_1$, and the information over channel 2 is used to identify a codevector $\lambda_2 \in \Lambda_2$, we will assume that the labeling function, α, is a mapping from Λ into $\Lambda_1 \times \Lambda_2$ and that $(\lambda_1, \lambda_2) = \alpha(\lambda)$. The component mappings are $\lambda_1 = \alpha_1(\lambda)$ and $\lambda_2 = \alpha_2(\lambda)$. Note that in order to decode to λ when both channels work, it is necessary that α be one-to-one.

Given Λ, Λ_1, Λ_2 and α, there are three distortions and two rates associated with the quantizer. For a given x mapped to the triple $(\lambda, \lambda_1, \lambda_2)$, the *two-channel distortion* d_0 is given by $\|x - \lambda\|^2$, the side distortions d_i by $\|x - \lambda_i\|^2$, $i = 1, 2$ (where $||\mathbf{x}||^2 \stackrel{\text{def}}{=} (1/L) \sum_{i=1}^{L} x_i^2$, is dimension-normalized). The corresponding average distortions are denoted by $\bar{d}_0$, $\bar{d}_1$ and $\bar{d}_2$. We assume that an entropy coder is used in order to transmit the labeled vectors at a rate arbitrarily close to the entropy, i.e., $R_i = \mathcal{H}(\alpha_i(Q(\mathbf{X})))/L$, $i = 1, 2$, where $\mathcal{H}$ is the entropy of its argument. The problem is to design the labeling function α so as to minimize $\bar{d}_0$ subject to $\bar{d}_1 \leq D_1$ and $\bar{d}_2 \leq D_2$, given rates (R_1, R_2) and distortions D_1 and D_2.

We will assume that the source is memoryless with probability density function (pdf) p. The L-fold pdf will be denoted by p_L where $p_L((x_1, x_2, \ldots, x_L)) = \prod_{i=1}^{L} p(x_i)$. Clearly, the differential entropies satisfy the relation $h(p_L) = Lh(p)$.

Given a lattice Λ and a sublattice Λ_1, and a point $\lambda_1 \in \Lambda_1$, let $V_{\Lambda_1 : \Lambda}(\lambda_1)$ denote the set of all points in Λ that are closer to λ_1 than to any other point in Λ_1. V is referred to as the Voronoi set of λ_1 in Λ. We will also use $V_\Lambda(\lambda)$ to denote the set of all points in $\mathbb{R}^L$ that are closer to λ than to any other point in Λ. The set $\mathcal{E}(\lambda_s) = \alpha(V_{\Lambda_s : \Lambda}(\lambda_s))$, $\lambda_s \in \Lambda_s$ is used to denote the set of all labels for points in $V_{\Lambda_s : \Lambda}(\lambda_s)$.

A Distortion Computation

The average two-channel distortion $\bar{d}_0$ is given by

$$\bar{d}_0 = \sum_{\lambda \in \Lambda} \int_{V_\Lambda(\lambda)} \|x - \lambda\|^2 p_L(x) dx. \tag{1}$$

Since the codebook of the quantizer is a lattice, all the Voronoi sets in the above summation are congruent. Furthermore, upon assuming that each Voronoi region is small and upon letting ν denote the L-dimensional volume of a Voronoi region, we obtain the two-channel distortion

$$\bar{d}_0 = \frac{\int_{V_\Lambda(0)} \|x\|^2 dx}{\nu} = G(\Lambda)\nu^{2/L}, \tag{2}$$

where the normalized second moment $G(\Lambda)$, defined by

$$G(\Lambda) = \frac{\int_{V_\Lambda(0)} \|x\|^2 dx}{\nu^{1+2/L}}. \tag{3}$$

When only description i is available, $i = 1, 2$, the distortion is given by

$$\bar{d}_i = \bar{d}_0 + \sum_{\lambda \in \Lambda} \|\lambda - \alpha_i(\lambda)\|^2 P(\lambda), \tag{4}$$

where $P(\lambda)$ is the probability of lattice point λ, and we have assumed that λ is the *centroid* of its Voronoi region. This is true for the uniform density. For nonuniform densities, there is an error term which goes to zero with the size of the Voronoi region. The first term in (4) is the two-channel distortion and the second term is the excess distortion which is incurred when only description i is available. Note that for a given Λ, only the excess distortion term is affected by the labeling α.

At this point we impose a constraint on the labeling function that allows us to reduce the problem to that of labeling a finite number of points. Specifically, we assume that the labeling function satisfies the property $\alpha(\lambda + \lambda_s) = \alpha(\lambda) + \lambda_s$, $\forall \lambda_s \in \Lambda_s$. This leads to the following simplification for the distortion:

$$\bar{d}_i = \bar{d}_0 + (1/N_s) \sum_{\lambda \in V_{\Lambda_s:\Lambda}(0)} \|\lambda - \alpha_i(\lambda)\|^2, \tag{5}$$

where we have assumed that $P(\lambda)$ is approximately constant over a Voronoi region of the sublattice Λ_s (but may vary from one Voronoi region to another).

B Rate Computation

Let R_0 (in bits/sample) be the rate required to address the two-channel codebook for a single channel system[1]. We will first derive an expression for R_0 and then determine the rates R_1 and R_2. We use the fact that each quantizer bin has identical volume ν and that $p_L(x)$ is approximately piecewise constant over each Voronoi region of Λ_1 and Λ_2. This assumption is valid in the limit as the Voronoi regions become small and is standard in asymptotic quantization theory.

The rate $R_0 = \mathcal{H}(Q(X))$ is given by

$$\begin{aligned} R_0 &= -(1/L) \sum_{\lambda} \int_{V_\Lambda(\lambda)} p_L(x)dx \log_2 \int_{V_\Lambda(\lambda)} p_L(x)dx \\ &\approx -(1/L) \sum_{\lambda} \int_{V_\Lambda(\lambda)} p_L(x)dx \log_2 p_L(\lambda)\nu \\ &\approx h(p) - (1/L) log_2(\nu). \end{aligned} \tag{6}$$

[1]This quantity is useful for evaluating the two-channel distortion as well as for evaluating the rate overhead associated with the multiple description system.

It can be shown that the rate for description i is given by

$$R_i = R_0 - (1/L)\log_2(N_i). \tag{7}$$

A single channel system would have used R_0 bits/sample. Instead a multiple description system uses a total of $R_1 + R_2 = 2R_0 - (1/L)\log_2(N_1 N_2)$ bits/sample, and so the rate overhead is $R_0 - (1/L)\log_2(N_1 N_2)$.

III Construction of the Labeling Function

In order to construct a labeling function, we first identify $\mathcal{E}$, the subset of $\Lambda_1 \times \Lambda_2$ each element of which will be a label for one point in Λ. Next a one-to-one correspondence will be established between $V_{\Lambda_s:\Lambda}(0)$ and a proper subset of $\mathcal{E}$, so as to minimize a suitable objective function, while ensuring that the labeling can be extended uniquely to the entire lattice. To this end we first start by formulating a cost criterion that will be used in the design.

A Cost criterion

The multiple descriptions problem is formulated [3] as minimizing the central distortion subject to constraints on the side distortion. Therefore, we can form the Lagrangian cost criterion given by

$$\begin{aligned} J &= \bar{d}_0 + \sum_i \gamma_i \bar{d}_i \qquad (8) \\ &= (\gamma_1 + \gamma_2 + 1)\bar{d}_0 + \sum_i \gamma_i \sum_{\lambda \in \Lambda} \|\lambda - \alpha_i(\lambda)\|^2 P(\lambda) \\ &= (\gamma_1 + \gamma_2 + 1)\bar{d}_0 + \sum_{\lambda \in \Lambda} P(\lambda) \sum_i \gamma_i \|\lambda - \alpha_i(\lambda)\|^2. \end{aligned}$$

In the lattice quantizer the distortion $\bar{d}_0$ is governed by the lattice Λ. If we use the assumption that $P(\lambda)$ is approximately constant over the Voronoi region of Λ_s, we can re-write the cost criterion in terms of the cost over a Voronoi region of Λ_s. Therefore our design problem reduces to finding a labeling scheme $\alpha(\lambda)$ which minimizes

$$\frac{1}{N_s} \sum_{\lambda \in V_{\Lambda_s:\Lambda}(0)} \left[\gamma_1 \|\lambda - \alpha_1(\lambda)\|^2 + \gamma_2 \|\lambda - \alpha_2(\lambda)\|^2\right]. \tag{9}$$

Now, by some fairly simple algebra, one can show that

$$\begin{aligned} &\gamma_1 \|\lambda - \alpha_1(\lambda)\|^2 + \gamma_2 \|\lambda - \alpha_2(\lambda)\|^2 = \qquad (10) \\ &\quad \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \|\alpha_2(\lambda) - \alpha_1(\lambda)\|^2 + (\gamma_1 + \gamma_2) \|\lambda - \frac{\gamma_1 \alpha_1(\lambda) + \gamma_2 \alpha_2(\lambda)}{\gamma_1 + \gamma_2}\|^2. \end{aligned}$$

Informally, our design principle is to find a labeling function $\alpha(\lambda)$ such that the two points $(\alpha_1(\lambda) \in \Lambda_1, \alpha_2(\lambda) \in \Lambda_2)$ are not very far apart, and such that the lattice point it labels is not very far from the weighted mean of these two sublattice points. This general guiding principle leads to our lattice design. We will first describe the basic quantizer design and then illustrate it with an example using the lattice $\mathbb{Z}^2$.

B Lattice Quantizer

The quantizer construction is based on the following steps.

1. We are given a lattice Λ, the rates R_1, R_2 and the distortions D_1, D_2. These determine the indices N_1, N_2, using (7), and we choose sublattices Λ_1, Λ_2 with these indices, and also the common sublattices Λ_s, Λ_{lcm}. We also choose the appropriate weights γ_1 and γ_2. For example for a successively refinable quantizer, the distortion D_2 is infinite and hence the re-use index N_2 is also infinite. By appropriately choosing $N_1, N_2, \gamma_1, \gamma_2$, one can achieve different levels of asymmetry in rate and distortion.

2. Identify the Voronoi region $V_0 = V_{\Lambda_s:\Lambda}(0)$ of the sublattice Λ_s. This is the fundamental region that we will label. The labeling is then extended to the full lattice using the shift invariance property (see Section II). We identify the sets $\mathcal{P}_1 = V_{\Lambda_s:\Lambda_1}(0)$, $\mathcal{P}_2 = V_{\Lambda_s:\Lambda_2}(0)$ which represent the points in Λ_i which lie in the Voronoi region.

3. We identify the set $\mathcal{L}_1(\lambda_1) = \{\lambda_2 \in \Lambda_2 : \lambda_2 \in V_0 + \lambda_1\}$ for all $\lambda_1 \in \mathcal{P}_1$. This represents the points in the sublattice Λ_2 which are in the set of points found by shifting the Voronoi region V_0 of Λ_s to center around the point $\lambda_1 \in \mathcal{P}_1$. This ensures that the edge length $(\|\alpha_2(\lambda) - \alpha_1(\lambda)\|^2)$ is minimized for our construction. We can show that each member of $\mathcal{L}_1(\lambda_1)$ lies in a different coset with respect to the sublattice shifts in Λ_s. Similarly we identify the set $\mathcal{L}_2(\lambda_2) = \{\lambda_1 \in \Lambda_1 : \lambda_1 \in V_0 + \lambda_2\}$ for all $\lambda_2 \in \mathcal{P}_2$. The set of edges emanating from V_0 are given by $\{(\lambda_1, \lambda_2) : \lambda_1 \in \mathcal{P}_1$ and $\lambda_2 \in \mathcal{L}_1(\lambda_1)$ OR $\lambda_2 \in \mathcal{P}_2$ and $\lambda_1 \in \mathcal{L}_2(\lambda_2)\}$. We find the coset leaders in the above edge set (relative to Λ_s) for our labelling scheme.

4. Matching the edges to the lattice points in the Voronoi region is posed as an assignment problem [18] and can be solved easily. To formulate the assignment problem we compute the cost given in (9) for each lattice point and an edge class (*i.e.* the edge and its Λ_s sub-lattice shifts). This is found by minimizing the cost over the edge class. To ensure shift invariance constraint, we allow only one coset member from the edge set to label a point in V_0.

   For lower computational complexity in the design, we can label only points in the sublattice Λ_{lcm} and use shift invariance on this sublattice. We can show that we do not lose any performance by doing so. Therefore, in the above procedure we replace the sets $\mathcal{P}_1, \mathcal{P}_2$ by the sets $\mathcal{P}_1^{'}, \mathcal{P}_2^{'}$ where $\mathcal{P}_1^{'} = V_{\Lambda_{lcm}:\Lambda_1}(0)$, and $\mathcal{P}_2^{'} = V_{\Lambda_{lcm}:\Lambda_2}(0)$. Also, we find the edge classes and coset leaders in the edge set with respect to the Λ_{lcm} sublattice. The rest of the procedure remains identical.

C Example

In this section we illustrate the design procedure with an example in two dimensions using the Z_2 lattice. We choose $|\Lambda_1| = 5$ and $|\Lambda_2| = 9$. The two sublattices are shown in Figure 2 with (Λ_1 : blue circle, Λ_2 : red cross, Λ_s : both blue circles and red crosses). There are 45 points in the Voronoi region V_0 of Λ_s. The set $\mathcal{P}_1$ contains 9 blue points and the set $\mathcal{P}_2$ contains 5 red points. The edges emanating from the Voronoi region of the sublattice Λ_s are shown. This is drawn using the sets $\{\mathcal{L}_1(\lambda_1)\}$ and $\{\mathcal{L}_2(\lambda_2)\}$. For example, if we take the blue point $\lambda_1 = (2,1)$ which is in $\mathcal{P}_1$, we see that it has 5 points in the set $\mathcal{L}_1(\lambda_1) = \{(0,0),(0,3),(3,3),(6,0),(3,0)\}$. Therefore, we identify the edge set by finding $\{\mathcal{L}_i(\lambda_i)\}$. We notice that there are several edges emanating from V_0 which are a sublattice Λ_s shift apart. For example the edge $\{(-2,-1),(-6,0)\}$ is a sublattice Λ_s shift away from the edge $\{(4,2),(0,3)\}$. To satisfy the shift invariance constraint, we would need only one of these edges to label a point in V_0, and this constraint is built into the optimization problem. The result of this optimization is illustrated in Figure 3. Here we have only drawn points in V_0 with each point having a label pair associated with it. The points in Λ_1, Λ_2 which label points in V_0 (written λ_1, λ_2) are also shown. The points in Λ_1 are labeled in blue and those in Λ_2 are labeled in red. In this example $\gamma_1 = 9$ and $\gamma_2 = 5$, which determines the respective distortions $\bar{d}_i$ obtained by the design. A comparison of these distortions with that predicted by information theory is given in Section V.

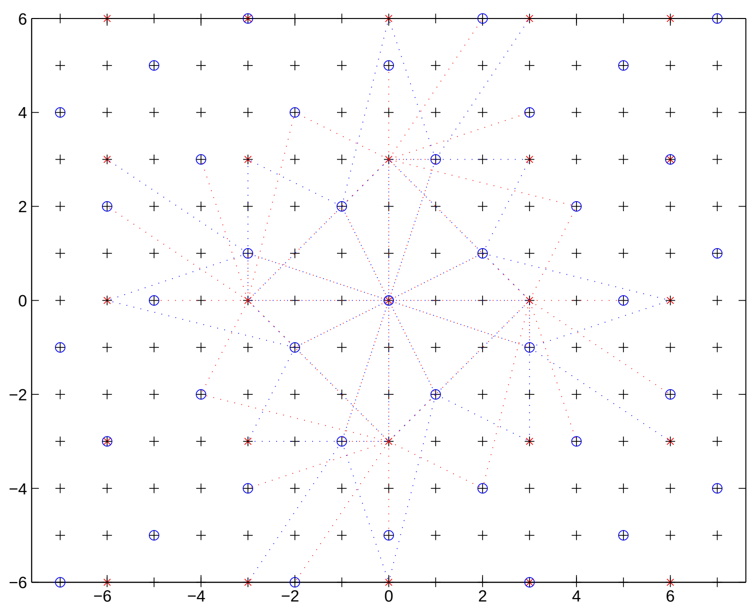

Figure 2: Edges emanating from the Voronoi region of the sublattice.

IV High rate asymptotics

In this section we analyze the high rate distortion behavior of the asymmetric multiple description lattice quantizer. We assume that there is a entropy coder after the quantizer and therefore evaluate the entropy constrained high rate bound [19]. We

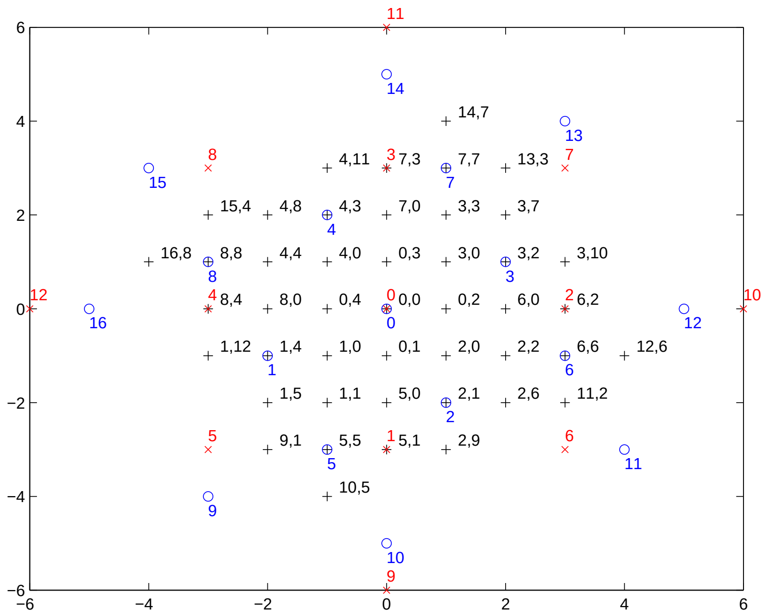

Figure 3: Labels generated by the algorithm.

first evaluate the behavior of the Lagrangian $\gamma_1\bar{d}_1 + \gamma_2\bar{d}_2$ and then find the approximation for $\bar{d}_i, i = 1, 2$. The latter would also allow us to predict the asymmetry in the distortion behavior of the quantizer.

Using the high rate approximation we can show that the Lagrangian is approximated by

$$\gamma_1\bar{d}_1 + \gamma_2\bar{d}_2 \approx \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\frac{1}{12}2^{2h(p)}2^{-2(R_1+R_2-R_0)} + (\gamma_1+\gamma_2)\frac{1}{12}2^{2(h(p)-R_0)}, \tag{11}$$

where R_0 determines the central distortion $\bar{d}_0$ and is given by $\bar{d}_0 = \frac{1}{12}2^{2(h(p)-R_0)}$. Moreover, this is written for the square lattices. For other lattices the factor of $\frac{1}{12}$ is replaced by the shaping gain for the lattice. This approximation is derived by using (8) and deriving a lower and upper bound based on it. The lower bound and the upper bound coincide asymptotically with the rate to the approximation given in (11). The side distortions are approximated by

$$\begin{aligned} \bar{d}_1 &\approx \frac{\gamma_2^2}{(\gamma_1+\gamma_2)^2}\frac{1}{12}2^{2h(p)}2^{-2(R_1+R_2-R_0)} + \frac{1}{12}2^{2(h(p)-R_0)} \\ \bar{d}_2 &\approx \frac{\gamma_1^2}{(\gamma_1+\gamma_2)^2}\frac{1}{12}2^{2h(p)}2^{-2(R_1+R_2-R_0)} + \frac{1}{12}2^{2(h(p)-R_0)} \end{aligned} \tag{12}$$

Note that though it appears as though the side distortions decay at the same rate, this need not be so. This is because γ_1, γ_2 could also be chosen to vary exponentially giving us different decay rates for the side distortions. Though this approximation is asymptotic in the rate, we observed that it was quite good for the quantizer design illustrated in the numerical results.

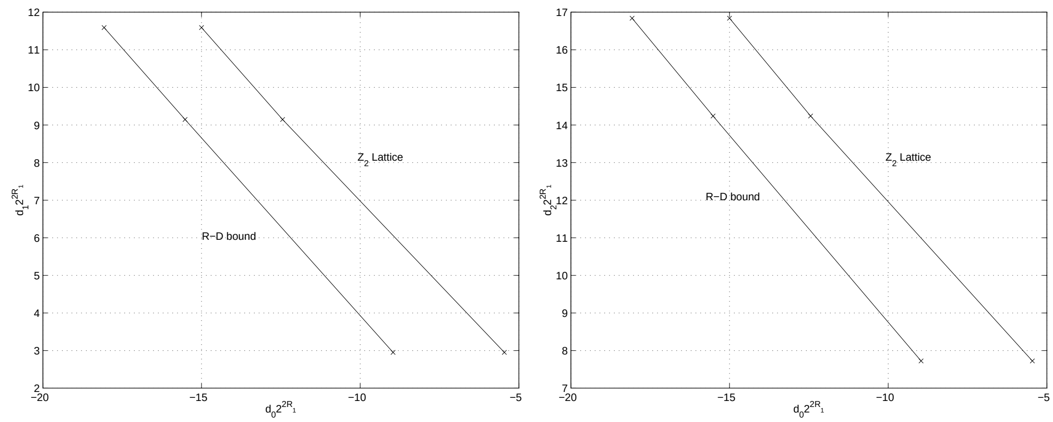

Figure 4: Comparison of lattice distortion with rate-distortion bound.

V Numerical Results

In order to illustrate the performance of the proposed quantizer, we evaluate its rate-distortion performance. In order to compare its performance with that predicted by information theory, we assume that there is an entropy (lossless) coding of the quantizer output. This is done for a Gaussian source with unit variance, for which the multiple description rate-distortion problem was solved by Ozarow [2].

The example chosen is the $\mathbb{Z}^2$ lattice that we described in Section III. The rates are chosen so that $R_1 - R_2 = \frac{1}{2}\log_2(\frac{|\Lambda_2|}{|\Lambda_1|})$. In Figure 4 we have plotted the side distortions and compared them with those predicted by information theory [2]. The key observation is that the distortion performance of the lattice quantizer is approximately 3dB away from that predicted by the rate-distortion bound. This gap is due to the shaping gain that we will pick up when we go to higher dimensions and using sublattices which have Voronoi regions which are close to spherical. The $\mathbb{Z}^2$ lattice used in this example is more for illustrative purposes and has very little shaping gain.

References

[1] M. Fleming and M. Effros, "Generalized multiple description vector quantization," in *Proceedings, Data Compression Conference*, pp. 3–12, March 29-31 1999.

[2] L. Ozarow, "On a source coding problem with two channels and three receivers," *Bell Syst. Tech. J.*, vol. 59, pp. 1909–1921, December 1980.

[3] A. A. El Gamal and T. M. Cover, "Achievable rates for multiple descriptions," *IEEE Trans. Inform. Th.*, vol. IT-28, pp. 851–857, November 1982.

[4] J. K. Wolf, A. D. Wyner, and J. Ziv, "Source coding for multiple descriptions," *Bell Syst. Tech. J.*, vol. 59, pp. 1417–1426, October 1980.

[5] Z. Zhang and T. Berger, "New results in binary multiple descriptions," *IEEE Trans. Inform. Th.*, vol. IT-33, pp. 502–521, July 1987.

[6] W. H. R. Equitz and T. M. Cover, "Successive refinement of information," *IEEE Trans. Inform. Th.*, vol. 37, pp. 269–275, March 1991.

[7] V. A. Vaishampayan, "Design of multiple description scalar quantizers," *IEEE Trans. Inform. Theory*, vol. 39, pp. 821–834, May 1993.

[8] V. Vaishampayan, "Vector quantizer design for diversity systems," in *Proc. Twenty-fifth Annual Conference on Information Sciences and Systems*, pp. 564–569, Johns Hopkins University, March 20–22 1991.

[9] S. D. Servetto, V. A. Vaishampayan, and N. J. A. Sloane, "Multiple description lattice vector quantization," in *Proceedings, Data Compression Conference*, pp. 13–22, March 29-31 1999.

[10] H. Jafarkhani and V. Tarokh, "Multiple description trellis coded quantizers," *IEEE Trans. Commun*, vol. 47, pp. 799–803, June 1999.

[11] A. E. Mohr, E. A. Riskin, and R. E. Ladner, "Graceful degradation over packet erasure channels through forward error correction," in *Proceedings, Data Compression Conference*, pp. 92–101, March 29-31 1999.

[12] P. A. Chou, S. Mehrotra, and A. Wang, "Decoding of overcomplete expansions using projections onto convex sets," in *Proceedings, Data Compression Conference*, pp. 72–81, March 29-31 1999.

[13] V. K. Goyal, J. Kovacevic, and M. Vetterli, "Quantized frame expansions as source-channel codes for erasure channels," in *Proceedings, Data Compression Conference*, pp. 326–335, March 29-31 1999.

[14] R. Balan, I. Daubechies, and V. A. Vaishampayan, "Trading rate for distortion through varying the redundancy of a windowed fourier frame in a multiple description compression." preprint, 1999.

[15] A. Ingle and V. A. Vaishampayan, "DPCM system design for diversity systems with applications to packetized speech," *IEEE Transactions on Speech and Audio Processing*, vol. 1, pp. 48–58, January 1995.

[16] M. Orchard, Y. Wang, V. Vaishampayan, and A. Reibman, "Redundancy rate distortion analysis of multiple description coding using pairwise correlating transforms," in *Proceedings of the 1997 International Conference on Image Processing*, Oct. 1997.

[17] V. Vaishampayan and A. A. Siddiqui, "Speech predictor design for diversity communication systems," in *Proceedings of the 1995 IEEE Speech Coding Workshop*, 20-22 September 1995.

[18] D. G. Luenberger, *Linear and Non-linear programming*. Reading, Mass.: Addison-Wesley, 2nd ed., 1984.

[19] R. M. Gray, *Source Coding Theory*. Massachusetts: Kluwer Academic Publishers, 1990.