Sequences and Pancakes

Many sequences arise in geometric problems, such as finding the largest number of pancake pieces that can be made by n straight cuts: 2, 4, 7, 11, 16, 22. . . . The sequence is generated by \(2^{n(n+1)} + 1\).

He has worked in coding theory, algebra and combinatorics. Perhaps his most intense interest and his best contribution to hard-core mathematicians lie in the area of “sphere packing” — a surprisingly large field of study devoted to questions of how best to arrange many identical balls so that they take up the least volume.

Sphere packing leads to some hard questions. In three dimensions, there is an obvious, very good arrangement: the regular, symmetrical array used for piling oranges or cannibals. Many crystals, at the molecular level, favor the same array. But mathematicians have never managed to prove that some other arrangement would not be even denser.

Where sphere packing really gets

lively, though, is in imaginary spaces of more dimensions than the usual three. Higher dimensions have a lot of room to play around in.

For example, the “kissing number” — the number of spheres that can be arranged around one central sphere — rises rapidly. In two dimensions, it is 6, as anyone can quickly see by placing some pennies on a table. In three dimensions, it is 12, but there is room left over, and some mathematicians long thought that there might somehow be a way to squeeze in 13th.

Dr. Sloane keeps handfuls of pennies and ball bearings within easy reach. His most important contributions to sphere packing have been clever ideas about spaces of 8 and 24 dimensions; realms where the mind truly boggles.

Almost to his disappointment, the elegant geometries of sphere packing in higher dimensions have given rise to practical applications. Communications engineers, for example, are developing strategies for efficiently transmitting the greatest possible information in a given “bandwidth,” and they are engaged in sphere packing. They want to squeeze in as many bits as possible, yet for the sake of clear communication they must keep them a certain distance apart.

In 24 dimensions, as it happens, the kissing number is 196,560 — part of a sequence that Dr. Sloane would add to his handbook, if only more terms were known for exact.

Mathematicians have tried similar catalogues of real numbers, for those whose mission is to identify pi the cube root of 12, but such things tend not to be quite as useful. “There are just too many real numbers,” as Dr. Sloane said.

Number sequences are special, somehow. Although in principle they are infinitely numerous, in reality the interesting ones seem relatively few. They capture a kind of logic about the world — a flexible, not quite cut-and-dried logic — which may be why designers of intelligence tests have always had a weakness for them, rightly or wrongly.

The logic of sequences can be tricky. The Sloane handbook lists no less than 22 different sequences that begin 1, 2, 3, 4, 5, . . . But a longer lead-in — Dr. Sloane generally insists on 10 numbers — almost always suffices to specify a sequence uniquely.

One reason is that sequences so often correspond to some simple question about geometry or combinatorics — how many ways objects can branch or fold or slice or combine. There are sequences for knots, trees, graphs and beads on necklaces. Sometimes finding the next member of such a sequence is a famous unsolved problem, yet the sequence is clearly measuring something fundamental.

So if you tell Dr. Sloane that you have come across a sequence in some interesting physical or mathematical context and that it begins 2, 4, 8, 15, 26, 42, 84, 53, he will say heavy odds that the next term will be 130. You have rediscovered a sequence that happens to be the greatest number of pieces you can get with successive slices through a cake.

Your problem may have nothing to do with cake; it may have nothing to do with geometry at all, as far as you can tell. No matter. When nature organizes itself into sequences, it seems to be a creature of habit.