In A `Random World,' He Collects Patterns, By James Gleick (New York Times, Tuesday January 27, 1997, Pages C1, C5)

Without quite meaning to, Neil J. A. Sloane has become the world's clearinghouse for number sequences.

He keeps track of easy ones, like 1, 2, 4, 8, 16, 32 . . ., the powers of two. He keeps track of hard ones, like 1, 1, 2, 5, 14, 38, 120, 353 . . ., the number of different ways of folding ever-longer strips of postage stamps. Among his sequences are the famous and the obscure: perfect, amicable and lucky numbers; Fibonacci, tribonacci and tetranacci numbers.

When a physicist stumbles upon a sequence he cannot explain, or a computer scientist creates a sequence he thinks is new, he writes to Dr. Sloane, a mathematician at AT&T Bell Laboratories in Murray Hill, N.J. Dr. Sloane then retrieves it from, or adds it to, his master list, now numbering around 5,000 different sequences.

It is a pursuit with an element of whimsy, as he readily admits. But it also reaches toward the heart of something that gives mathematics its universality. Over and over, the same sequences pop up in many different contexts - one day in number theory, the next day in solid-state physics - demonstrating unsuspected connections in nature.

"It emphasizes the fact that what mathematics is looking for is patterns in a world that really looks rather random," said Richard K. Guy of the University of Calgary, editor of the Unsolved Problems department of The American Mathematical Monthly and a frequent contributor to Dr. Sloane's collection. "Often these sequences tie things together which you wouldn't normally tie together."

So Dr. Sloane's mail is heavy - recent letters have come from places as various as Finland, the Philippines and Kalamazoo. His life has been taken over by numbers that start to seem as familiar as faces. To mathematicians and scientists working in seemingly distant fields, Dr. Sloane has become a strange and useful pipeline.

N. J. A. Sloane: Enclosed find a sequence of numbers, single digit, for which I am seeking a fraction or formula. . . . Read horizontally: 9, 3, 2, 4, 8, 9, 1, 1, 6, 5, 8, 3, 7, 1 . . . Any charge for this, please let me know.

Like so many of his correspondents, Dr. Sloane got interested in number sequences by coming across one he could not reduce to a formula, in the 1960's, when he was a graduate student at Cornell University. The sequence began with 1, 8, 78, 944 . . . "I remember the 944 very well," he said the other day, pacing energetically in his box-strewn office.

More and more, such sequences turn up in applied sciences. Physicists studying the behavior of molecules in solid lattices, for example, produce sequences by adding up all the possible paths through a regular geometric space, and that makes it a problem of combinatorics.

It is natural for any scientist, trying to understand the rules governing combinations of many things, to see what happens with one thing, then two, then three . . . and so create a number sequence. If it is a sequence that mathematicians already understand, he is in luck.

So Dr. Sloane began to assemble a book. Many papers dealing with the properties of numbers or the different ways of combining or arranging objects listed a few sequences. He spent hours searching libraries and entering numbers on computer punch cards. When other mathematicians heard about the project, they contributed sequences by the dozens.

"A Handbook of Integer Sequences" appeared in 1973 and became an instant classic, at least by the extremely finite standards of mathematics books. Unfortunately, it also became instantly outdated.

New sequences poured in. Now Dr. Sloane's collection is fully computerized, and he is planning a new edition, although many sequences still lie uncollated in cartons on the floor. In theory, of course, the number of sequences is infinite. "The book could get very thick indeed," he said. "I've had to draw a line and throw away sequences that I considered not so interesting."

He has certain rules. Sequences must consist entirely of whole numbers. They must be infinitely long, which disqualifies the famous sequence 14, 18, 23, 28, 34, 42, 50, 59 . . . - the local stops on the West Side IRT.

For some sequences, a simple formula calculates any desired term. To get the nth number in the sequence of perfect squares (1, 4, 9, 16 . . .), just multiply n by itself. For other sequences, the process is less direct - one calculates the nth term from the terms immediately before it, using a rule known as a "recurrence." Among the best-known sequences generated by a recurrence is the Fibonacci sequence, in which each term is the sum of the two preceding.

For many other sequences, though, neither a formula nor a recurrence is known, even when they are easy to define. An example is the Mersenne primes - prime numbers that are one less than a power of 2, such as 3, 7, 31. Finding the next Mersenne prime quickly starts to tax the limits of computational ingenuity. It is not even known for certain whether they meet the requirement of being an infinite sequence, but Dr. Sloane gives them "the benefit of the doubt." And sequences can be defined in many conceivable ways, not all of which get them into the book. There are some mind-twisting possibilities. "You couldn't have the first sequence that's not in the book," said Ronald L. Graham, head of mathematical research at Bell Labs.

Dear Neil, Let me try to kill two birds with one sloane. Here is a new sequence: 1, 2, 3, 8, 10, 54, 42, 944, 5112 . . .

As the curator of this bizarre museum, Dr. Sloane, a wiry native of Wales who is also a serious rock climber, sometimes regrets the time lost to more earnest mathematics. "I probably spend too much time on it," he said. "I should have done more abstract things."

He has worked in coding theory, algebra and combinatorics. Perhaps his most intense interest and his best contribution to hard-core mathematics lie in the area of "sphere packing" - a surprisingly large field of study devoted to questions of how best to arrange many identical balls so that they take up the least volume.

Sphere packing leads to some hard questions. In three dimensions, there is an obvious, very good arrangement: the regular, symmetrical array used for piling oranges or cannonballs. Many crystals, at the molecular level, favor the same array. But mathematicians have never managed to prove that some other arrangement would not be even denser.

Where sphere packing really gets lively, though, is in imaginary spaces of more dimensions than the usual three. Higher dimensions have a lot of room to play around in.

For example, the "kissing number" - the number of spheres that can be arranged around one central sphere - rises rapidly. In two dimensions, it is 6, as anyone can quickly see by placing some pennies on a table. In three dimensions, it is 12, but there is room left over, and some mathematicians long thought that there might somehow be a way to squeeze in a 13th.

Dr. Sloane keeps handfuls of pennies and ball bearings within easy reach. His most important contributions to sphere packing have been clever ideas about spaces of 8 and 24 dimensions, realms where the mind truly boggles.

Almost to his disappointment, the elegant geometries of sphere packing in higher dimensions have given rise to practical applications. Communications engineers, for example, devising strategies for efficiently transmitting the greatest possible information in a given "bandwidth," find that they are engaged in sphere packing. They want to squeeze in as many bits as possible, yet for the sake of clean communication they must keep them a certain distance apart.

In 24 dimensions, as it happens, the kissing number is 196,560 - part of a sequence that Dr. Sloane would add to his handbook, if only more terms were known for sure.

Dear Dr. Sloane, . . . It would seem to me that since the Mega Test is a "take home" test, a person could find all the "series" answers on that test just by consulting your book. I threw away that stupid test, so I can't check to see whether it's true. . . .

Mathematicians have tried similar catalogues of real numbers, for those who quickly need to identify pi [or] the cube root of 12, but such things tend not to be quite so useful. "There are just too many real numbers," as Dr. Sloane said.

Number sequences are special, somehow. Although in principle they are infinitely numerous, in reality the interesting ones seem relatively few. They capture a kind of logic about the world - a flexible, not quite cut-and-dried logic - which may be why designers of intelligence tests have always had a weakness for them, rightly or wrongly.

The logic of sequences can be tricky. The Sloane handbook lists no less than 22 different sequences that begin 1, 2, 3, 4, 5 . . . But a longer lead-in - Dr. Sloane generally insists on 10 numbers - almost always suffices to specify a sequence uniquely.

One reason is that sequences so often correspond to some simple question about geometry or combinatorics - how many ways objects can branch or fold or slice or combine. There are sequences for knots, trees, graphs and beads on necklaces. Sometimes finding the next member of such a sequence is a famous unsolved problem, yet the sequence is clearly measuring something fundamental.

So if you tell Dr. Sloane that you have come across a sequence in some interesting physical or mathematical context and that it begins 2, 4, 8, 15, 26, 42, 64, 93, he will lay heavy odds that the next term will be 130. You have rediscovered a sequence that happens to be the greatest number of pieces you can get with successive slices through a cake. Your problem may have nothing to do with cake; it may have nothing to do with geometry at all, as far as you can tell. No matter. When nature organizes itself into sequences, it seems to be a creature of habit. [Minor errors in transcript corrected by N. J. A. Sloane, April 26 2023]