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On the Existence of a Projective Plane of Order 10

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If a projective plane of order 10 exists, let $\mathcal{A}$ denote the (111, 56) binary error-correcting code generated by the rows of the incidence matrix. It is known that the weight distribution of this code is uniquely determined by the number of codewords of weights 12, 15 and 16. It is the object of this paper to report that the number of codewords of weight 15 is zero, thus reducing the number of unknown weights to two. Part of this calculation was carried out by computer.

1. INTRODUCTION

It is not known if a projective plane of order 10 exists (see for example [4]). We shall assume here that one does exist, and study the properties it must have.

A projective plane of order 10 is an arrangement of 111 points and 111 lines with the following properties:

(1.1) Each line contains 11 points, and each point meets 11 lines.

(1.2) Any two distinct points lie on exactly one line, and any two distinct lines meet in exactly one point.

Let $\chi = (\chi_{ij})$ be the 111×111 incidence matrix of the plane, where $\chi_{ij} = 1$ if line l_i contains point p_j, and 0 otherwise.

Then each row of χ has weight 11. (By the weight $|v|$ of a vector v we mean the number of non-zero components.)

Let F be the field GF(2). Let $\mathcal{A}$ be the subspace of F^{111} generated by the rows of χ. $\mathcal{A}$ may be thought of as a binary error-correcting code of block length 111. Let w_i be the number of vectors of $\mathcal{A}$ of weight i. The numbers $\{w_i\}$ provide a good deal of information about the plane.

It is known (see [1] and [10]) that the numbers $\{w_i\}$ are uniquely determined if we know w_{12}, w_{15}, w_{16}. It is the object of this paper to report that $w_{15} = 0$, thus reducing the number of unknowns to two.

Some earlier applications of electronic computers to the study of finite projective planes will be found in [2, 3, 5–9].

2. Properties of the Code $\mathcal{A}$

(Some of the results of this section have been given independently by Assmus [1].) It is convenient to define the extended code $\bar{\mathcal{A}}$. This is the subspace of F^{112} generated by the rows of the matrix $\bar{\chi}$ formed from χ by adding a column of 111 ones. Equivalently, $\bar{\mathcal{A}}$ is formed from $\mathcal{A}$ by annexing a 0 to all vectors of even weight, and a 1 to all vectors of odd weight. If $v \in \mathcal{A}$, $\bar{v}$ will denote the corresponding vector of $\bar{\mathcal{A}}$.

Let $\bar{\mathcal{A}}^{\perp}$ denote the orthogonal complement of $\bar{\mathcal{A}}$ in F^{112}.

It can be shown (see [10]) that $\bar{\mathcal{A}} = \bar{\mathcal{A}}^{\perp}$, but we need only the weaker result that

$$(2.1) \qquad \bar{\mathcal{A}} \subset \bar{\mathcal{A}}^{\perp}.$$

Proof. The rows of the extended matrix $\bar{\chi}$ all have weight 12, and by (1.2) distinct rows intersect in two points.

Therefore the generators of $\bar{\mathcal{A}}$ are all contained in $\bar{\mathcal{A}}^{\perp}$, hence $\bar{\mathcal{A}} \subset \bar{\mathcal{A}}^{\perp}$.

We think of vectors of $\mathcal{A}$ both as binary vectors of length 111, and as the corresponding subsets of points of the plane.

$$(2.2) \qquad \bar{v} \in \bar{\mathcal{A}} \Rightarrow |\bar{v}| \equiv 0 \pmod 4.$$

Proof. This is true for the generators of $\bar{\mathcal{A}}$ which all have weight 12. It is then true for all vectors of $\bar{\mathcal{A}}$ by the formula

$$|\bar{v} + \bar{w}| = |\bar{v}| + |\bar{w}| - 2\,|\bar{v} \cap \bar{w}|,$$

since $|\bar{v} \cap \bar{w}|$ is even by (2.1). From (2.2) follows

$$(2.3) \qquad v \in \mathcal{A} \Rightarrow |v| \equiv 0 \quad \text{or} \quad 3 \pmod 4.$$

(2.4) Let l be any line of the plane, and v any vector of $\mathcal{A}$. Then

$$|v \cap l| \equiv |v| \pmod 2.$$

Proof. By (2.1), $|\bar{v} \cap \bar{l}| \equiv 0 \pmod 2$. If v has even weight, then $|v| = |\bar{v}|$, and so

$$|v \cap l| = |\bar{v} \cap \bar{l}| \equiv 0 \pmod 2.$$

If v has odd weight,

$$|v \cap l| = |\bar{v} \cap \bar{l}| - 1 \equiv 1 \pmod 2.$$

(2.5) $\qquad v \in \mathcal{O}\!\!\mathcal{l}, \quad |v| \neq 0 \Rightarrow |v| \geqslant 11.$

Proof. (i) If $|v|$ is odd, let us look at the 11 lines through a point $p \notin v$. By (2.4), v must intersect each at least once, and so $|v| \geqslant 11$.

(ii) If $|v|$ is even and non-zero, consider the 11 lines through a point $p \in v$. Using (2.4) again, v must meet each line in at least one additional point, and so $|v| \geqslant 12$.

(2.6) The vectors of weight 11 in $\mathcal{O}\!\!\mathcal{l}$ are exactly the 111 lines of the plane.

Proof. (i) By definition all the lines are in $\mathcal{O}\!\!\mathcal{l}$.

(ii) Conversely let $|v| = 11$. Let l be a line through 2 points p, q of v. By (2.4), there is a third point r of v on l. If $v \neq l$, there is a fourth point $x \notin v$ on l. The 10 remaining lines through x must by (2.4) contain at least one point of v. Thus $|v| \geqslant 13$, a contradiction. Hence $v = l$.

DEFINITION. An *oval* of the plane is a set of 12 points, no 3 of which are collinear. (It is not known whether the plane, if it exists, has any ovals.)

(2.7) The vectors of weight 12 in $\mathcal{O}\!\!\mathcal{l}$ are exactly the ovals of the plane.

Proof. (i) Suppose $v \in \mathcal{O}\!\!\mathcal{l}$, $|v| = 12$. Let l be any line. By (2.4), $|v \cap l|$ is even, say $|v \cap l| = 2s$. Consider the 11 lines through one of these $2s$ points. One of them is l, and the other 10, by (2.4), must meet v at least once more. Hence

$$2s + 10 \leqslant 12,$$

and so either $2s = 2$ or $2s = 0$. This implies that v is an oval.

(ii) Conversely we show that every oval σ is in $\mathcal{O}\!\!\mathcal{l}$. Let S be the set of $\binom{12}{2} = 66$ lines through the pairs of points of σ. There are 11 lines of S

through each point of σ. Consider the sum over all points of the plane and all lines of S,

$$\sum_{p}\sum_{l\in S}\chi(p,l)=\sum_{p\in\sigma}\sum_{l\in S}\chi(p,l)+\sum_{p\notin\sigma}\sum_{l\in S}\chi(p,l)$$

$$=12\cdot 11+\sum_{p\notin\sigma}\sum_{l\in S}\chi(p,l).$$

But

$$\sum_{l\in S}\sum_{p}\chi(p,l)=66\cdot 11;$$

hence

$$\sum_{p\notin\sigma}\sum_{l\in S}\chi(p,l)=99\cdot 6.$$

Since $\sum_{l\in S}\chi(p,l)\leqslant 6$ for $p\in\sigma$, and the number of points not in σ is 99 this implies $\sum_{l\in S}\chi(p,l)=6$ for $p\notin\sigma$.

Now consider these 66 lines as generators of the code $\mathcal{A}$ and take their vector sum (over GF(2)). The result is the weight 12 vector corresponding to σ.

From the preceding it follows that

(2.8) $w_0=1$, $w_i=0$ for $i=1,\ldots,10$, $w_{11}=111$, and $w_{13}=w_{14}=0$.

It is further known (see [1] and [10]) that w_{12}, w_{15} and w_{16} uniquely determine the other w_i. We now show that $w_{15}=0$.

3. There are no Vectors of Weight 15

Let us suppose on the contrary that there is a vector of weight 15 in $\mathcal{A}$, and let $A=\{1,2,\ldots,15\}$ denote the corresponding set of 15 points.

(3.1) If l is any line, $|A\cap l|=1$, 3 or 5.

Proof. From (2.4), $|A\cap l|$ is odd and, since

$$|A+l|=15+11-2|A\cap l|\geqslant 12,$$

it follows $|A\cap l|\leqslant 7$. If $|A\cap l|=7$, 2.7 implies $A+l$ is an oval σ. But then $|l\cap\sigma|=4$, which contradicts the definition of an oval.

(3.2) Let b_i be the number of lines meeting A in exactly i points, $i=1,3,5$. Then

$$b_1=90,\qquad b_3=15,\qquad b_5=6.$$

Proof. From (3.1), $b_1 + b_3 + b_5 = 111$. Each pair of points of A determines one of these lines, so

$$\binom{3}{2} b_3 + \binom{5}{2} b_5 = \binom{15}{2}.$$

A third equation is obtained by counting the following sum in two ways:

$$\sum_{p \notin A} \sum_{l} \chi(l, p) = (111 - 15)\, 11 = \sum_{l} \sum_{p \notin A} \chi(l, p) = 10b_1 + 8b_3 + 6b_5 .$$

These equations are then solved for the b_i.

The First Six Lines

Let $B_1, \ldots, B_6$ be the six lines meeting A in 5 points.

(3.3) $$B_i \cap B_j \in A.$$

Proof. Suppose $B_i \cap B_j = p \notin A$. B_i and B_j together contain 10 points of A. The other 9 lines through p must each contain at least one more point of A, from 2.4, but there are only 5 points of A left—a contradiction.

(3.4) Any three or more B_i's do not have a common point.

The proof is straightforward and is omitted.

Thus each of the 15 points of A is uniquely a $B_i \cap B_j$, and so without loss of generality we may assume that they are arranged in the lines B_1–B_6 as follows:

B_1 :	1	2	3	4	5
B_2 :	1	6	7	8	9
B_3 :	2	6	10	11	12
B_4 :	3	7	10	13	14
B_5 :	4	8	11	13	15
B_6 :	5	9	12	14	15

FIGURE 1

These six lines cannot have any other intersections among themselves, so the remaining 36 points occurring on them must be distinct. Let $B = \{76, 77, \ldots, 111\}$ be this set of points, and without loss of generality let us arrange them as in Figure 3a.

The Fifteen Lines

We next consider the fifteen lines $C_1, C_2, \ldots, C_{15}$ (say) that meet A in exactly 3 points.

(3.5) Each point of A meets exactly 3 lines C_i.

Proof. A point $p \in A$ already occurs in the same line with 8 other points of A in Figure 1, leaving only 6 points of A as possible partners in the lines C_i (i.e., 3 lines C_i). Since there are 45 intersections between A and the C_i, each point of A occurs exactly thrice.

There is now essentially only one way to arrange the points of A in the lines C_1–C_{15}, as follows:

C_1 :	1	10	15
C_2 :	1	11	14
C_3 :	1	12	13
C_4 :	2	7	15
C_5 :	2	8	14
C_6 :	2	9	13
C_7 :	3	6	15
C_8 :	3	8	12
C_9 :	3	9	11
C_{10} :	4	6	14
C_{11} :	4	7	12
C_{12} :	4	9	10
C_{13} :	5	6	13
C_{14} :	5	7	11
C_{15} :	5	8	10

FIGURE 2

There remain $111 - 15 - 36 = 60$ points with which to fill the 120 empty places on the C_i. Let $C = \{16, 17, \ldots, 75\}$ be this set of points.

(3.6) Each of these 60 points meets exactly two lines C_i.

Proof. Let $p \notin A \cup B$, and suppose three lines C_i pass through p. This uses 9 points of A. In the remaining 90 lines p can occur only $15 - 9 = 6$ times. Hence p meets only $3 + 6 = 9$ lines, contradicting (1.1).

Since there are 120 places to fill, p must appear exactly twice, and all 60 remaining points are used.

The lines C_i may now be completed in an essentially unique way. Figure 3b shows the arrangement actually used.

	1	2	3	4	5	76	77	78	79	80	81
	1	6	7	8	9	82	83	84	85	86	87
(a)	2	6	10	11	12	88	89	90	91	92	93
	3	7	10	13	14	94	95	96	97	98	99
	4	8	11	13	15	100	101	102	103	104	105
	5	9	12	14	15	106	107	108	109	110	111

	1	10	15	46	47	52	53	58	59	64	65
	1	11	14	40	41	55	56	61	62	70	71
	1	12	13	43	44	49	50	67	68	73	74
	2	7	15	32	33	34	36	61	63	67	69
	2	8	14	25	26	38	39	58	60	73	75
	2	9	13	22	23	28	29	64	66	70	72
	3	6	15	29	30	37	38	49	51	56	57
(b)	3	8	12	16	17	35	36	46	48	71	72
	3	9	11	18	20	31	33	52	54	74	75
	4	6	14	22	24	34	35	43	45	53	54
	4	7	12	18	21	37	39	40	42	65	66
	4	9	10	16	19	26	27	55	57	68	69
	5	6	13	25	27	31	32	41	42	47	48
	5	7	11	17	19	28	30	44	45	59	60
	5	8	10	20	21	23	24	50	51	62	63

FIG. 3. Twenty-one lines of the plane.

The Remaining 90 *lines*

We have now determined 21 out of the 111 lines. The remaining 90 lines each meet A in exactly one point, and each point of A lies on 6 of these lines.

(3.7) Let l be any one of the 90 lines. Then $| l \cap B | = 4$ and $| l \cap C | = 6$.

Proof. Suppose $1 \in l$. Then l meets B_1 and B_2 in the point 1, and must meet B_3, B_4, B_5, B_6 in exactly four points taken from $(B_1 \cup B_2 \cup B_3 \cup B_4) \cap B$. Therefore $| l \cap B | = 4$. Also

$$| l \cap B | + | l \cap C | = 10.$$

A Group Fixing A

(3.8) The set A is invariant under a group G isomorphic to the symmetric group $\mathscr{S}_6$.

Proof. Let $\mathscr{S}_6$ act on the symbols t_1, t_2,..., t_6 and let τ_i denote conjugation by (t_i, t_{i+1}), $1 \leqslant i \leqslant 5$. Then τ_1,..., τ_5 generate a group G isomorphic to $\mathscr{S}_6$. Let us identify the points 1,..., 15 of A with the transpositions (t_i, t_j) as shown:

1	2	3	4	5	6	7	8
(t_1t_2)	(t_1t_3)	(t_1t_4)	(t_1t_5)	(t_1t_6)	(t_2t_3)	(t_2t_4)	(t_2t_5)

9	10	11	12	13	14	15
(t_2t_6)	(t_3t_4)	(t_3t_5)	(t_3t_6)	(t_4t_5)	(t_4t_6)	(t_5t_6)

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	τ_1	1	6	7	8	9	2	3	4	5	10	11	12	13	14	15
(a)	τ_2	2	1	3	4	5	6	10	11	12	7	8	9	13	14	15
	τ_3	1	3	2	4	5	7	6	8	9	10	13	14	11	12	15
	τ_4	1	2	4	3	5	6	8	7	9	11	10	12	13	15	14
	τ_5	1	2	3	5	4	6	7	9	8	10	12	11	14	13	15

		16	17	18	19	20	21	22	23	24	25	26	27	28	29
	τ_1	21	18	17	20	19	16	25	27	26	22	24	23	31	32
	τ_2	18	20	16	21	17	19	43	44	45	41	40	42	50	49
	τ_3	26	25	22	27	23	24	18	20	21	17	16	19	31	33
	τ_4	18	21	16	20	19	17	29	28	30	32	33	31	23	22
	τ_5	20	18	17	21	16	19	25	26	27	22	23	24	39	38

		30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
	τ_1	33	28	29	30	38	39	37	36	34	35	71	70	72	73	74
	τ_2	51	48	47	46	53	54	52	57	56	55	26	25	27	22	23
	τ_3	32	28	30	29	37	39	38	34	36	35	43	44	45	40	41
	τ_4	24	27	25	26	38	37	39	35	34	36	46	47	48	49	50
	τ_5	37	35	34	36	32	31	33	30	29	28	44	43	45	41	40

		45	46	47	48	49	50	51	52	53	54	55	56	57	58	59
	τ_1	75	65	64	66	67	68	69	59	58	60	62	61	63	53	52
	τ_2	24	33	32	31	29	28	30	36	34	35	39	38	37	61	63
(b)	τ_3	42	58	59	60	61	62	63	64	65	66	68	67	69	46	47
	τ_4	51	40	41	42	43	44	45	55	56	57	52	53	54	61	62
	τ_5	42	52	53	54	56	55	57	46	47	48	50	49	51	64	65

		60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
	τ_1	54	56	55	57	47	46	48	49	50	51	41	40	42	43	44	45
	τ_2	62	58	60	59	67	69	68	64	66	65	73	75	74	70	72	71
	τ_3	48	49	50	51	52	53	54	56	55	57	74	73	75	71	70	72
	τ_4	63	58	59	60	70	71	72	73	74	75	64	65	66	67	68	69
	τ_5	66	67	68	69	58	59	60	61	62	63	73	74	75	70	71	72

FIG. 4. Action of group G on sets A and C.

Thus G acts on A. Figure 4a shows the action of the generators on A.

The lines B_1–B_6 may be identified with

$$\{(t_1t_2), (t_1t_3), (t_1t_4), (t_1t_5), (t_1t_6)\}$$

and its images under conjugation, and the lines C_1–C_{15} identified with

$$\{(t_1t_2), (t_3t_4), (t_5t_6)\}$$

and its images under conjugation. Then

(3.9) G acts as a collineation on the lines B_1–B_6 and on the lines C_1–C_{15}.

Since each point of C is the intersection of some two lines C_i and C_j, G can be extended to act on C. Figure 4b shows the action of the generators on C.

Finally, let G_1 be the subgroup of G which fixes the point 1. G_1 has order 48 and is generated by τ_1, τ_3, τ_4, τ_5.

The Remaining 90 *Lines Cannot Be Completed*

We now describe the method by which a computer was used to show that the remaining 90 lines cannot be completed.

(I) First we consider those six lines (out of the 90) which meet A in point 1. Clearly the 36 points of C on these lines must be selected from the numbers which are not crossed out in Figure 5.

Further the six points of C on such a line must account for the intersections of that line with the 12 lines remaining in Figure 5. Thus we must

~~1~~	~~10~~	~~15~~	~~46~~	~~47~~	~~52~~	~~53~~	~~58~~	~~59~~	~~64~~	~~65~~	
~~1~~	~~11~~	~~14~~	~~40~~	~~41~~	~~55~~	~~56~~	~~61~~	~~62~~	~~70~~	~~71~~	
~~1~~	~~12~~	~~13~~	~~43~~	~~44~~	~~49~~	~~50~~	~~67~~	~~68~~	~~73~~	~~74~~	
~~2~~	~~7~~	~~15~~	32	33	34	36	~~61~~	63	~~67~~	69	32
~~2~~	~~8~~	~~14~~	25	26	38	39	~~58~~	60	~~73~~	75	60
~~2~~	~~9~~	~~13~~	22	23	28	29	~~64~~	66	~~70~~	72	22
~~3~~	~~6~~	~~15~~	29	30	37	38	~~49~~	51	~~56~~	57	51
~~3~~	~~8~~	~~12~~	16	17	35	36	~~46~~	48	~~71~~	72	16—6-set
~~3~~	~~9~~	~~11~~	18	20	31	33	~~52~~	54	~~74~~	75	18
~~4~~	~~6~~	~~14~~	22	24	34	35	~~43~~	45	~~53~~	54	22
~~4~~	~~7~~	~~12~~	18	21	37	39	~~40~~	42	~~65~~	66	18
~~4~~	~~9~~	~~10~~	16	19	26	27	~~55~~	57	~~68~~	69	16
~~5~~	~~6~~	~~13~~	25	27	31	32	~~41~~	42	~~47~~	48	32
~~5~~	~~7~~	~~11~~	17	19	28	30	~~44~~	45	~~59~~	60	60
~~5~~	~~8~~	~~10~~	20	21	23	24	~~50~~	51	~~62~~	63	51

FIG. 5. Constructing list L_1.

find a set of 6 numbers such that exactly one occurs in each line of Figure 5. An example is shown in the last column of Figure 5.

It was found that there are exactly 344 such 6-sets. Let L_1 denote the set of these 344 6-sets.

Similarly let L_i be the 344 6-sets which are candidates for the points of C on the six lines which meet A in point i only, for $i = 2,..., 15$. L_2–L_{15} may be obtained from L_1 by applying the appropriate elements of G.

On the six lines through 1 we need 6 disjoint 6-sets from L_1. We call such an object a K_6. The set of all K_6's can be arranged in orbits under G_1, and it is clearly sufficient to obtain just one representative from each orbit.

A simple backtracking program was used to find all the representative K_6's. There were exactly 1021. These were punched out on cards for subsequent use. (The running time for this part of the program was about 3 minutes.) The term K_6 is chosen because of the method used. A graph is constructed in which the nodes represent the 344 6-sets of L_1, and two nodes are joined by an edge if and only if the corresponding 6-sets are disjoint. A set of 6 disjoint 6-sets is then represented by a complete graph on 6 nodes (i.e., a K_6).

	1	88	94	100	106
	1	89	95	101	107
(a)	1	90	96	102	108
	1	91	97	103	109
	1	92	98	104	110
	1	93	99	105	111
	10	76	82	100	107
	10	77	83	101	108
(b)	10	78	84	102	109
	10	79	85	103	110
	10	80	86	104	111
	10	81	87	105	106

FIG. 6. Showing the arrangement of the points C on the 90 lines.

Let U be the K_6 chosen to go on the six lines through 1. These lines must now be filled out with 24 points from B, in fact, points which lie on the lines B_3,..., B_6. Without loss of generality these may be chosen as shown in Figure 6a. We now have 27 lines of the projective plane.

(II) We next try to fill in the 6 lines which meet A in point 10 only; the points of C on these lines must be a K_6 from L_{10}. (*N.B.*—point 10 is considered next because the intersection of L_1 and L_{10} is greater than the intersection of L_1 and L_i, $i = 2,..., 9$. This reduces computation time and storage requirements.)

A sample arrangement of points of B on the six lines through 10 is shown in Figure 6. This of course is not a "general" arrangement; however, we can say that each line through 10 meets exactly 2 lines through 1 in a point of B, and hence must meet the other 4 in a point of C. Thus an admissible 6-set from L_{10} must miss two 6-sets of U and meet the other 4 in exactly one point each.

For each 6-set in L_{10} we therefore define a "matching vector"

$$\mathbf{m} = m_1, m_2, ..., m_6$$

where m_i is the cardinality of its intersection with the i-th row of U. We eliminate from L_{10} all 6-sets for which the matching vector does not

1	17	18	24	26	29	32
1	16	21	28	31	34	38
1	23	27	30	33	35	39
1	19	20	22	25	36	37
1	45	48	57	63	66	75
1	42	51	54	60	69	72
10	30	36	42	43	70	75
10	33	37	41	45	72	73
10	18	25	28	35	56	67
10	17	22	31	39	49	61
10	29	34	40	48	60	74
10	32	38	44	54	66	71
11	21	27	29	36	53	73
11	16	23	32	37	43	58
11	34	39	47	50	57	72
11	35	38	42	63	64	68
11	22	26	48	51	65	67
11	24	25	46	49	66	69
15	17	21	25	54	68	70
15	16	18	22	41	50	60
15	19	31	35	62	66	73
15	20	28	39	43	48	55
15	24	27	40	44	72	75
15	23	26	42	45	71	74

FIG. 7. A set of 24 partial lines.

consist of 4 ones and 2 zeros. (For each of the 1021 choices of U, the number of survivors from L_{10} was always less than 100.)

We now find all sets V of 6 disjoint 6-sets from the reduced L_{10}. The number of these for a particular U varied between 0 and 80.

(III) For each choice of U and V we next consider the 6 lines which meet A in point 15 only. For any possible arrangement of the B points each of these lines meet exactly two lines through 1 and two lines through 10 in a point of B. Each 6-set in L_{15} has two matching vectors, m, m', and we eliminate those for which either one is not correct. We now find all sets W of 6 disjoint 6-sets from the reduced L_{15}.

In only 98 of the original 1021 K_6's were any sets W found. The maximum number of sets W corresponding to any pair U, V was 10.

(IV) For each acceptable triple U, V, W we try to find a compatible X of 6 disjoint 6-sets chosen from L_{11}. A total of 25 acceptable quadruples U, V, W, X were found. One such set of 24 partial lines is displayed in Figure 7.

(V) However none of these can be extended to include a fifth set of 6 disjoint 6-sets from L_{14}.

We conclude that there is no way to construct the set of 90 lines, and therefore our initial hypothesis that a vector of weight 15 exists is false.

The total running time for this part of the program was just under 3 hours. The computer used was a General Electric 635.

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