SIAM J. DISC. MATH.
Vol. 2, No. 1, pp. 28-37, February 1989

003

CODES FROM SYMMETRY GROUPS, AND A [32, 17, 8] CODE*

YING CHENG† AND N. J. A. SLOANE‡

Abstract. Let G be the automorphism group of the four-dimensional cube, a group of order $2^4 \cdot 4! = 384$. The binary codes associated with the 32-dimensional permutation representation of G on the edges of the cube are investigated. There are about 400 such codes, one of which is a [32, 17, 8] code, having twice as many codewords as the [32, 16, 8] extended quadratic residue code.

Key words. codes, error-correcting codes, permutation groups, modular representations

AMS(MOS) subject classifications. 94B, 20C

1. Introduction. Since random codes are good ([1], [27, p. 558], [29]), one wishes to identify families of codes large enough to have a chance of including some good codes, yet small enough to be manageable. In this paper we describe one such family: the codes obtained from the action of the automorphism group of the n-dimensional cube on its m-dimensional faces.

In particular, the automorphism group G of the four-dimensional cube, a group of order 384, permutes the 32 edges of that cube. Regarding the edges as a basis, we have a 32-dimensional vector space V over $GF(2)$ on which G acts. The codes we consider are the subspaces of V invariant under G. There are about 400 such subspaces, one of which is a [32, 17, 8] binary code.

We find this quite astonishing, since the well-known second-order Reed–Muller and extended quadratic-residue codes of length 32 are [32, 16, 8] codes, and are extremal Type II self-dual codes ([16], [17, p. 194], [24]). It is remarkable that there should be a linear code with the same minimal distance and twice as many codewords. Of course the new code is not self-dual. Its properties are summarized in Theorem 1.

This family of codes can be generalized in several ways. Besides varying the dimensions of the cube and the faces, we could consider other regular polytopes instead of the cube, or more generally other Weyl groups (our group G is the Weyl group of type B_4).

Many other codes have been obtained from modular representations of groups in the past. Of course classical cyclic codes arise from the regular representations of cyclic groups, and include a large number of good examples. In the 1960s Berman [4], [5], Camion [11], Delsarte [19], and MacWilliams [25], [26] studied other abelian groups, but (perhaps because of the limitations of the computers available) did not find any especially interesting codes.

In 1975 Lomonaco (see [15]) found a record [45, 13, 16] binary code obtained as an invariant subspace of the regular representation of the group $C_3 \times C_{15}$. In [10], Calderbank and Wales found a [176, 22, 50] code from the Higman–Sims simple group. Brooke [7]–[9] has studied a large number of other simple groups, using Parker's "meat-axe" [28], without, however, finding any new record codes. Representation theory has also been used to construct codes by Liebler [23], Camion [12], Rabizzoni [32], Ward [34], Zlotnik [36], Klemm [21], Charpin [13], [14], Bhattacharya [6], Jensen [20], Wolfmann [35], and Landrock and Manz [22].

However, it seems fair to say that our [32, 17, 8] binary code is the first record code of length less than 100 that comes from a *modular* representation (where the characteristic

* Received by the editors January 21, 1988; accepted for publication July 8, 1988.

† Department of Mathematics, Louisiana State University, Baton Rouge, Louisiana 70803. Present address, AT&T Bell Laboratories, Holmdel, New Jersey 07733.

‡ Mathematical Sciences Research Center, AT&T Bell Laboratories, Murray Hill, New Jersey 07974.

of the field divides the order of the group). Furthermore, in contrast to many of the papers mentioned, we do not use the *regular* representation of the group. Another distinguishing feature of our approach is the relatively large number of invariant subspaces that occur, increasing the chance that one of them is good!

2. The new code. Let G be the automorphism group of the four-dimensional cube, a group of structure $2^4.S_4$ and order $2^4 \cdot 4! = 384$. This group permutes the 32 edges of the cube, which we label as in Fig. 1. Let V be a 32-dimensional vector space over $GF(2)$ with basis that is in one-to-one correspondence with the edges, so that G acts on V. A typical vector $v \in V$ has the form $v = (v_1, \cdots, v_{32})$, $v_i = 0$ or 1, with coordinates corresponding to the labels in Fig. 1. We write these vectors in hexadecimal notation, with 0 = 0000, $\cdots$, 9 = 1001, A = 1010, $\cdots$, F = 1111. We may also identify v with the corresponding set of edges.

Any set of vectors $u, v, \cdots \in V$ generates a binary linear code of length 32, denoted by $\langle u, v, \cdots \rangle$, namely the modulo-2 span of the union of the orbits of $u, v, \cdots$ under G. These codes are the G-invariant subspaces of V. A code or subspace $\langle u \rangle$ with a single generator is called *cyclic*, following [18, p. 52]. (This is the appropriate generalization of the standard term from coding theory.)

We denote the G-invariant codes of dimension k by $C_k = C_k^{(0)}, C_k^{(1)}, \cdots$, and when they are cyclic we denote corresponding generating vectors by $u_k = u_k^{(0)}, u_k^{(1)}, \cdots$. The labels are chosen so that, for $k \neq 16$, $C_k^{(i)}$ and $C_{32-k}^{(i)}$ are dual codes. Also $C_{16}^{(2i)}$ and $C_{16}^{(2i+1)}$ are duals $(0 \leqq i \leqq 2)$. We shall make use of the particular generators $u_k^{(i)}$ shown in Table 1. Some generators that represent geometrically interesting configurations in the cube are displayed in Fig. 2.

The code C_{17} is the most interesting, and we summarize its properties in the following theorem.

THEOREM 1. *The code* $C_{17} = \langle u_{13}, u_{14} \rangle$ *is a* [32, 17, 8] *binary code, with generator matrix as shown in Fig.* 3(a). (*An alternation definition is given in* § 3.) *This code has the following weight distribution*:

i	0	8	10	12	14	16
A_i	1	908	3328	14784	27392	38246

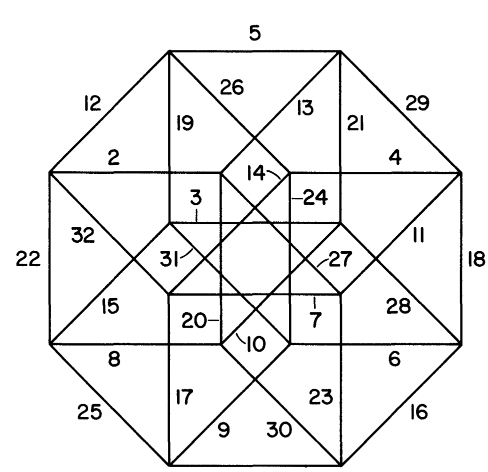

FIG. 1. *Four-dimensional cube with the* 32 *edges labeled.*

TABLE 1
Generating vectors $u_k^{(i)}$ for selected code $C_k^{(i)}$ (in hexadecimal).

u_1	FFFFFFFF	$u_9^{(9)}$	111111EE
u_3	00FF00FF	$u_9^{(10)}$	0F0F5A5A
u_4	FF000000	$u_9^{(11)}$	0F0F5AA5
u_5	F0F0F0F0	u_{11}	00335A69
$u_5^{(1)}$	CC99CC99	$u_{11}^{(1)}$	11117822
u_6	55AA5555	$u_{11}^{(2)}$	003C3C5A
$u_6^{(1)}$	00665533	$u_{11}^{(3)}$	111E111E
$u_6^{(2)}$	006655CC	$u_{11}^{(4)}$	11224B78
u_7	0F695A3C	$u_{11}^{(6)}$	00005A3C
$u_7^{(1)}$	3C693C69	u_{13}	AAA50000
$u_7^{(2)}$	000F00F0	$u_{13}^{(1)}$	1111444B
$u_7^{(3)}$	33663C69	$u_{13}^{(2)}$	030648E7
$u_7^{(4)}$	1E2D4B78	$u_{13}^{(3)}$	03068481
$u_7^{(5)}$	00335566	$u_{13}^{(4)}$	11111E1E
$u_7^{(7)}$	1E2D4B87	$u_{13}^{(5)}$	03060306
$u_7^{(8)}$	11224477	$u_{14}^{(1)}$	03770605
$u_7^{(9)}$	88112244	$u_{14}^{(2)}$	0F184184
$u_7^{(10)}$	0F3C3C5A	$u_{14}^{(3)}$	03090CCA
$u_7^{(11)}$	0F3C3CA5	u_{16}	03091242
u_9	0F0F3C69	$u_{16}^{(2)}$	03116050
$u_9^{(1)}$	0F3C5A69	$u_{16}^{(4)}$	00174184
$u_9^{(2)}$	003C5569	u_{27}	88840000
$u_9^{(3)}$	33693369	u_{28}	00008200
$u_9^{(4)}$	1E1E1E1E	u_{29}	80808040
$u_9^{(7)}$	1E1E1EE1	u_{31}	08100000
$u_9^{(8)}$	11111111		

with $A_{32-i} = A_i$, and G is its full automorphism group. The covering radius of C_{17} is 6, *a typical deep hole being* **00001117** *(in hexadecimal). The dual code is $C_{15} = \langle u_{15} \rangle$, a* [32, 15, 8] *code with generator matrix as shown in Fig.* 3(b), *and has the following weight distribution*:

i	0	8	10	12	14	16
A_i	1	124	1152	3584	6016	11014

with $A_{32-i} = A_i$. All G-invariant subcodes of C_{17} and C_{15} are as shown in Figs. 4 *and* 5; *in particular C_{17} and C_{15} intersect in the* [32, 9, 8] *code $C_9^{(5)}$. The double circles in Figs.* 4 *and* 5 *show all the cyclic modules in these diagrams; C_{17} itself is not cyclic.*

Remarks. (i) The dual lattice to Fig. 5 gives all the codes containing C_{17}.

(ii) The best way to remember these codes is to notice that the generator u_{15} for the dual C_{15} resembles two umbrellas, one of which has lost its fabric (see Fig. 2). This vector is stabilized by a subgroup of G of order six.

(iii) In Table 1 we give more than enough generators to enable any of the codes in Figs. 4 and 5 or their duals to be reconstructed. (The Bensen and Conway [3] notion of reduced lattice of submodules was helpful in preparing Table 1.) For completeness we note that G itself is generated by the following permutations:

$$(1,15,17,8,9,22)(2,16,19,7,10,24)(3,14,20,6,12,23)$$

$$(4,13,18,5,11,21)(26,27,28)(30,31,32)$$

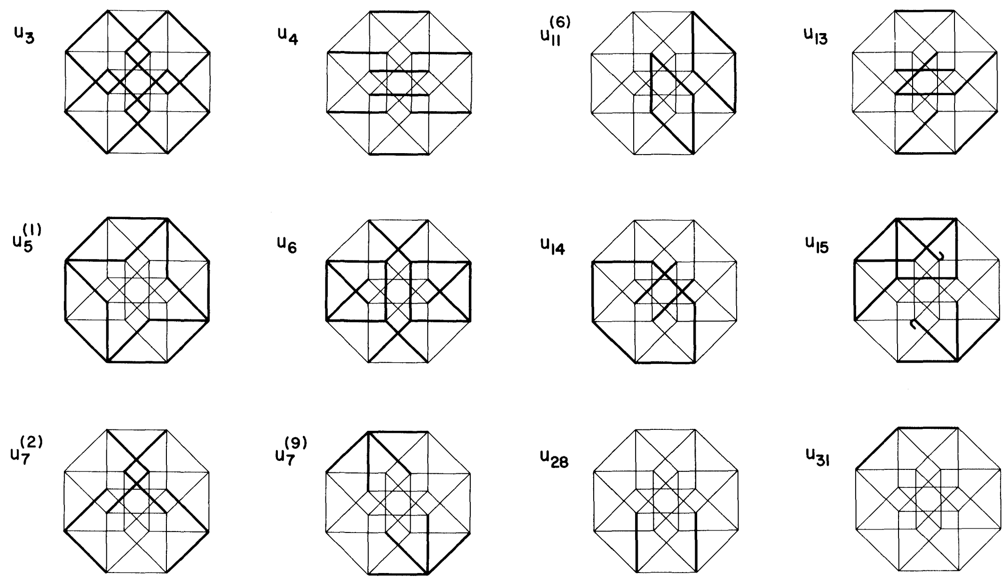

FIG. 2. *Subsets of edges corresponding to selected generating vectors.*

(a)

10101010101010010100000000000000
01101010111000010000011010100000
00110000000100010000011000000101
0001000100000011000000010100110000
00001010011001010100010000000110
00000110001000010010010000001100
00000011000100010110111110100000
00000000101010101010010100000000
00000000010101011010010100000000
00000000001111000101101001100110
00000000000101110001010000100001
00000000000011110110011000110011
00000000000000001111111100000000
00000000000000000101010101011010
00000000000000000011001101100110
00000000000000000000111100001111
00000000000000000000000011111111

(b)

11101000000110110010111001000100
010001001110100000001101100101110
00100010010000101101011100101110
00010001101100101110010010001011
00001001000011000110111100111111
00000110101010010011010111001111
00000011000010010011111101010011
00000000100110011010101000110011
00000000011001101010101011001100
00000000001111001001011000110011
00000000000011110101010100001111
00000000000000001111111100000000
00000000000000000101101000111100
00000000000000000011001101100110
00000000000000000000000011111111

FIG. 3. *Generator matrices for codes* (a) C_{17} *and* (b) C_{15}.

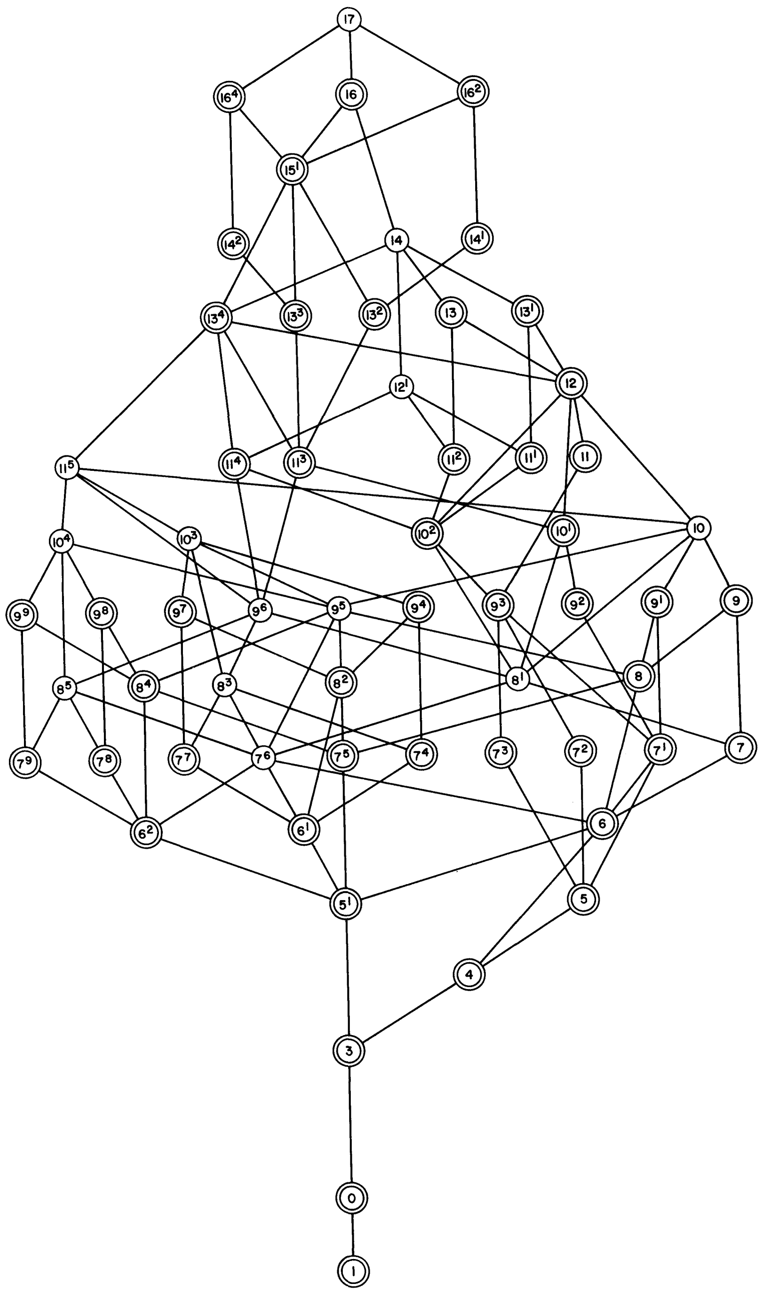

FIG. 4. *Complete lattice of G-invariant subcodes of* C_{17}. *The code* $C_k^{(i)}$ *is abbreviated* k^i *in Figs.* 4 *and* 5. *Cyclic modules (with one generator) are indicated by double circles.*

and

$$(1,9,17,25)(2,10,18,26)(3,11,19,27)(4,12,20,28)$$

$$(5,13,21,29)(6,14,22,30)(7,15,23,31)(8,16,24,32).$$

(iv) The following list identifies, from the set of codes mentioned in Figs. 4 and 5 and their duals, all those that have minimal distance $d \geqq 6$:

$d=6$: $C_6, C_{16}^{(3)}, C_{17}^{(1)}, C_{18}, C_{18}^{(2)}$;

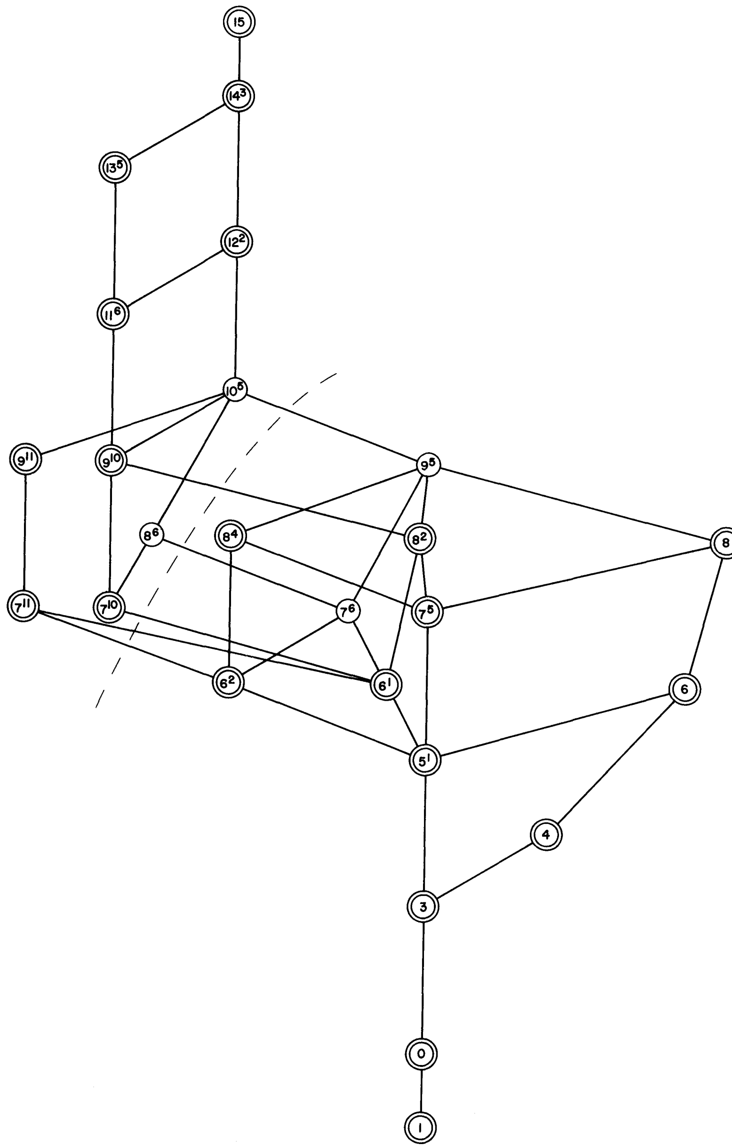

FIG. 5. *Complete lattice of G-invariant subcodes of* C_{15}.

$d=8$: $C_4, C_5, C_7^{(i)}\,(i=0,\cdots,3,6,9), C_8^{(i)}\,(i=0,\cdots,6), C_9^{(i)}\,(i=0,\cdots,11),$
$C_{10}^{(i)}\,(i=0,\cdots,5), C_{11}^{(i)}\,(i=0,\cdots,6), C_{12}^{(i)}\,(i=0,1,2), C_{13}^{(i)}\,(i=0,\cdots,5),$
$C_{14}^{(i)}\,(i=0,\cdots,3), C_{15}, C_{15}^{(1)}, C_{16}^{(i)}\,(i=0,\cdots,5), C_{17};$

$d=12$: $C_6^{(1)}, C_6^{(2)}, C_7^{(i)}\,(i=4,5,7,8,10,11);$

$d=16$: $C_3, C_5^{(1)};$

$d=32$: C_1.

(v) A dense 32-dimensional lattice sphere packing may be obtained from C_{17} by applying Construction D of [2]. This packing (see [17, p. 235]) has center density $\delta =$

2 and each sphere touches 249,280 others, and is the second densest packing known in this dimension. (Quebbemann's 32-dimensional lattice ([30], [31], [17, p. 220]) has $\delta = 2.566 \cdots$ and each sphere touches 261,120 others.)

The group $G = 2^4.S_4$, like S_4, has just two conjugacy classes of elements of odd order, and so, again like S_4, has just two absolutely irreducible representations over $GF(2)$ (cf. [18, p. 58]). These are the trivial one-dimensional representation and the two-dimensional representation by the following matrices of $GL_2(2)$:

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \right\}. \tag{1}$$

THEOREM 2. (a) *Every composition series of V begins and ends:*

$$\{0\} = C_0 \subset C_1 \subset \cdots \subset C_{31} \subset C_{32} = V,$$

where $C_1 = \{0^{32}, 1^{32}\}$ *and* C_{31} *consists of all even-weight vectors. In particular, every nontrivial G-invariant code is even, contains* 1^{32}, *and its weight distribution satisfies* $A_i = A_{32-i}$.

(b) *One composition series for V is*

$$\begin{aligned} \{0\} &= C_0 \subset C_1 \subset C_3 \subset C_4 \subset C_6 \subset C_7 \subset C_9 \subset C_{10} \\ &\subset C_{12} \subset C_{13} \subset C_{14} \subset C_{16} \subset C_{17} \subset C_{18}^{(3)} \subset C_{20}^{(2)} \subset C_{22}^{(5)} \\ &\subset C_{23}^{(5)} \subset C_{24} \subset C_{26} \subset C_{28} \subset C_{29} \subset C_{31} \subset C_{32} = V. \end{aligned}$$

(c) *The composition factors for V are* $1^{12}2^{10}$.

Before proving these theorems we describe what we think is the full list of invariant subspaces.

CONJECTURE. (a) The complete list of G-invariant subcodes of V consists of 373 codes, whose dimensions k are as follows:

k	1	2	3	4	5	6	7	8
#	1	0	1	1	2	3	14	16
k	9	10	11	12	13	14	15	16
#	20	16	19	16	17	22	22	31

(The number of dimension $32 - k$ is equal to the number of dimension k.)

(b) The code C_{17} is the unique G-invariant code of minimal distance $d \geqq 8$ and dimension $k \geqq 17$. The largest G-invariant codes of minimal distances 4, 6, 12, 16 have dimensions 25, 18, 8, 5, respectively (and are not especially good; cf. Verhoeff [33]).

(c) There are nine self-dual codes, all with minimal distance $d = 2$ or 4 (e.g., the vectors **00000011**, **0000000F** generate self-dual codes with $d = 2, 4$, respectively). The nontrivial Reed–Muller, extended Hamming, and extended quadratic-residue codes of length 32 are not G-invariant codes.

Remark. The 373 codes described in (a) (and in Figs. 4 and 5) are only claimed to be distinct, not necessarily inequivalent. But usually distinct G-invariant codes are inequivalent. More precisely, if C and C' are equivalent codes (implying that there is a permutation $\pi \in S_{32}$ with $C^\pi = C'$) such that Aut (C) = Aut $(C') = G$, then $C = C'$. For Aut $(C') = \pi$ Aut $(C)\pi^{-1}$ = Aut $(C) = G$, implying that π is in the normalizer of G in S_{32}. But G is equal to its normalizer, so $\pi \in G$, and $C = C'$.

Proof of Theorem 1. The assertions about the dimension, weight distribution, covering radius, and dual code are routine computer verifications.

By definition, Aut $(C_{17}) \supseteq G$. To prove equality, we first examined (by computer) the weight distributions of the nonlinear subcode formed by the 908 codewords of weight

8. There are exactly four weight 8 codewords with weight distribution $A_0 = 1, A_8 = 180, A_{17} = 544, A_{16} = 183$, namely the vectors **FF000000, 00FF0000, 0000FF00, 000000FF**. (These are supported on the four classes of eight parallel edges of the cube, see Fig. 1.) Thus the division of the 32 coordinates into these four blocks of eight is canonical. The group G induces all 4! permutations of the four blocks.

There are exactly 28 codewords meeting the blocks $4 + 4 + 0 + 0$, and these have the form $(u, u, 0, 0)$ and $(u, \bar{u}, 0, 0)$, where u is a weight 4 word in an [8, 4, 4] Hamming code $\mathscr{H}_8$. The automorphism group of $\mathscr{H}_8$ has structure $2^3.L_2(7)$ [2, p. 399], and the permutations induced by G on the first block yield exactly the 2^3 part of this group. G also contains the permutation $(9, 10)(11, 12) \cdots (31, 32)$, fixing the blocks and fixing every point of the first block. Any permutation of C_{17} not in G can then be assumed to fix the blocks, and to act as an element of $L_2(7)$ inside each block. We verified by computer that all such permutations are already in G. Thus Aut $(C_{17}) = G$.

The assertion that Figs. 4 and 5 show all G-invariant subcodes of C_{17} and C_{15} was proved as follows. We first established what we believe is a complete list of all G-invariant subcodes of V. There are 373 codes, as described above. (This list was constructed by a variety of techniques: repeatedly taking joins, intersections, and duals; constructing a generator matrix for each code and finding the cyclic module generated by each row; finding the cyclic modules generated by all vectors of selected codes; and other ad hoc methods.) The list was checked to be closed under the operations of taking joins, intersections, and duals. We examined the cyclic codes generated by every vector of C_{17} and C_{15}, and verified that these are on the list. This proves the assertion.

Proof of Theorem 2. (a) From the remarks preceding the theorem we know that the composition factors are all 1 or 2. Suppose a composition series begins $C_0 \subset C_2 \subset \cdots$, where C_2 is a two-dimensional code generated by vectors u, v and affording the two-dimensional representation (1). Then every $g \in G$ sends u to u, v or $u + v$, and all three occur. Since G is transitive, $|u \cap \bar{v}| + |u \cap v| + |\bar{u} \cap v| = 32$. Since u can be mapped to v, $|u \cap \bar{v}| = |\bar{u} \cap v|$, and similarly $|u \cap \bar{v}| = |u \cap v|$, so the three sets are equal in size and $3|u \cap v| = 32$, which is impossible. The assertion $\cdots C_{31} \subset C_{32}$ follows by duality.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a & a & 0 & 0 \\ b & b & 0 & 0 \\ c & c & 0 & 0 \\ a & 0 & a & 0 \\ b & 0 & b & 0 \\ c & 0 & c & 0 \\ a & 0 & 0 & a \\ b & 0 & 0 & b \\ c & 0 & 0 & c \\ 0 & w & w & w \\ x & 0 & x & x \\ y & y & 0 & y \\ z & z & z & 0 \end{bmatrix}$$

FIG. 6. *Alternative generator matrix for* C_{17} (0 = 00000000, 1 = 11111111).

(b), (c) The computer was used to verify that all the composition factors of 2 in the given series are irreducible.

3. An alternative construction. The [32, 17, 8] code C_{17} described in Theorem 1 was in fact first found by the following construction. This provides an alternative description, and may be of independent interest.

Let $\mathscr{H}_8$ and $\mathscr{H}'_8$ be two versions of the [8, 4, 4] Hamming code that intersect only in $\{0^8, 1^8\}$. (For example, take the point-code and line-code shown in [17, Fig. 11.27].) Choose independent vectors $a, b, c \in \mathscr{H}_8$ that span $\mathscr{H}_8/\{0^8, 1^8\}$, and vectors $w, x, y, z \in \mathscr{H}'_8$ that span $\mathscr{H}'_8/\{0^8, 1^8\}$ and satisfy $w + x + y + z = 0$. (For example, $a = 10101001$, $b = 10011100$, $c = 10000111$, $w = 11001100$, $x = 10101010$, $y = 11110000$, $z = 10010110$.) Then Fig. 6 generates a code equivalent to C_{17}. (It is not difficult to find an isomorphism onto the earlier version. The first four rows of Fig. 6 are the four special codewords mentioned in the proof of Theorem 1.)

Acknowledgments. We are grateful to John Conway and Walter Feit for some very helpful suggestions.

Note added in proof. Gerhard J. A. Schneider of the University of Essen has verified that Conjecture (a) is correct, using the CAYLEY computer system. There are indeed exactly 373 G-invariant subcodes. (Personal communication, June 10, 1988.)

REFERENCES

[1] R. Ahlswede and G. Dueck, *Good codes can be produced by a few permutations*, IEEE Trans. Inform. Theory, 27 (1981), pp. 398–408.
[2] E. S. Barnes and N. J. A. Sloane, *New lattice packings of spheres*, Canad. J. Math., 35 (1983), pp. 117–130.
[3] D. J. Benson and J. H. Conway, *Diagrams for modular lattices*, J. Pure Appl. Algebra, 37 (1985), pp. 111–116.
[4] S. D. Berman, *On the theory of group codes*, Kibernetika, 3, no. 1, (1967), pp. 31–39. (In Russian.) Cybernetics, 3, no. 1, (1967), pp. 25–31. (In English.)
[5] ———, *Semisimple cyclic and Abelian codes* II, Kibernetika, 3, no. 3, (1967), pp. 21–30. (In Russian.) Cybernetics, 3, no. 3, (1967), pp. 17–23. (In English.)
[6] P. Bhattacharya, *On a class of Abelian codes*, Inform. Sci., 33 (1984), pp. 173–179.
[7] P. L. H. Brooke, *Tables of codes associated with certain finite simple groups*, Ph.D. Dissertation, University of Cambridge, Cambridge, UK, June 1984.
[8] ———, *On matrix representations and codes associated with the simple group of order* 25920, J. Algebra, 91 (1984), pp. 536–566.
[9] ———, *On the Steiner system* $S(2, 4, 28)$ *and codes associated with the simple group of order* 6048, J. Algebra, 97 (1985), pp. 376–406.
[10] A. R. Calderbank and D. B. Wales, *A global code invariant under the Higman–Sims group*, J. Algebra, 75 (1982), pp. 233–260.
[11] P. Camion, *Abelian Codes*, Report 1059 Mathematics Research Center, University of Wisconsin, 1970.
[12] ———, *Etude des codes binaires Abelians modulaires autoduaux de petites longuers*, Rev. CETHEDEC, 1979, no. 2, pp. 3–24.
[13] P. Charpin, *The extended Reed–Solomon codes considered as ideals of a modular algebra*, Ann. Discrete Math., 17 (1983), pp. 171–176.
[14] ———, *A description of some extended cyclic codes with application to Reed–Solomon codes*, Discrete Math., 56 (1985), pp. 117–124.
[15] J. H. Conway, S. J. Lomonaco, Jr., and N. J. A. Sloane, *A* [45, 13] *code with minimal distance* 16, Discrete Math., to appear.
[16] J. H. Conway and V. Pless, *On the enumeration of self-dual codes*, J. Combin. Theory Ser. A, 28 (1980), pp. 26–53.
[17] J. H. Conway and N. J. A. Sloane, *Sphere Packings, Lattices and Groups*, Springer-Verlag, New York, 1988.
[18] C. W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, John Wiley, New York, 1962.
[19] P. Delsarte, *Automorphisms of Abelian codes*, Philips Res. Reports, 25 (1970), pp. 389–402.

[20] J. M. JENSEN, *The concatenated structure of cyclic and Abelian codes*, IEEE Trans. Inform. Theory, 31 (1985), pp. 788–793.

[21] M. KLEMM, *Kennzeichnung der erweiterten Quadrate-codes durch ihre PSL(2, q)-Zulässigkeit*, Commun. Algebra, 11 (1983), pp. 2051–2068.

[22] P. LANDROCK AND O. MANZ, *Classical codes as ideals in group algebras*, preprint.

[23] R. A. LIEBLER, *On codes in the natural representations of the symmetric group*, in Combinatorics, Representation Theory, and Statistical Methods in Groups, Marcel Dekker, New York, 1980.

[24] C. L. MALLOWS AND N. J. A. SLOANE, *An upper bound for self-dual codes*, Inform. and Control, 22 (1973), pp. 188–200.

[25] F. J. MACWILLIAMS, *Codes and ideals in group algebras*, in Combinatorial Mathematics and Its Applications, R. C. Bose and T. A. Dowling, eds., University North Carolina Press, Chapel Hill, NC, 1969, Chap. 18.

[26] ———, *Binary codes which are ideals in the group algebra of an Abelian group*, Bell System Tech. J., 49 (1970), pp. 987–1011.

[27] F. J. MACWILLIAMS AND N. J. A. SLOANE, *The Theory of Error-Correcting Codes*, North-Holland, Amsterdam, 1979.

[28] R. A. PARKER, *The computation of modular characters (the meat-axe)*, in Computational Group Theory, M. D. Atkinson, ed., Academic Press, New York, 1984, pp. 267–274.

[29] J. N. PIERCE, *Limit distribution of the minimum distance of random linear codes*, IEEE Trans. Inform. Theory, 13 (1967), pp. 595–599.

[30] H.-G. QUEBBEMANN, *A construction of integral lattices*, Mathematika, 31 (1984), pp. 137–140.

[31] ———, *Lattices with theta-functions for* $G(\sqrt{2})$ *and linear codes*, J. Algebra, 105 (1987), pp. 443–450.

[32] P. RABIZZONI, *Images binaires d'ideaux principaux d'une algèbre de groupe*, Rev. CETHEDEC, 1981, no. 2.

[33] T. VERHOEFF, *An updated table of minimum-distance bounds for binary linear codes*, IEEE Trans. Inform. Theory, 33 (1987), pp. 665–680.

[34] H. N. WARD, *Divisible codes*, Arch. Math., 36 (1981), pp. 485–494.

[35] J. WOLFMANN, *A group algebra construction of binary even self-dual codes*, Discrete Math., 65 (1987), pp. 81–89.

[36] B. M. ZLOTNIK, *Doubly transitive groups of type* $p^m(p^m - 1)$ *and maximal nonbinary codes generated by them*, Kibernetika, 19, no. 3, (1983), pp. 16–20. (In Russian.) Cybernetics, 19 (1983), pp. 309–316. (In English.)