JOURNAL OF NUMBER THEORY **48**, 373–382 (1994)

On Lattices Equivalent to Their Duals

J. H. CONWAY

Mathematics Department, Princeton University, Princeton, New Jersey 08544

AND

N. J. A. SLOANE

Mathematical Sciences Research Center, AT&T Bell Laboratories, Murray Hill, New Jersey 07974

Communicated by M. Waldschmidt

Received May 12, 1992; revised July 16, 1992

A lattice is called isodual if it is geometrically congruent to its dual. We show that the densest three-dimensional isodual lattice is the "mean centered-cuboidal" lattice, a lattice which is in a sense the mean of the face-centered and body-centered cubic lattices. This lattice is also the most economical three-dimensional isodual covering. We give a number of other dense isodual lattices in $\mathbb{R}^n$, $n \leqslant 24$.

1. INTRODUCTION

In a recent paper, Bergé and Martinet [3] say that a lattice Λ is *dual-extreme* if it achieves a local maximum of $\{\mu(\Lambda)\,\mu(\Lambda^*)\}^{1/2}$, where Λ^* is the lattice dual to Λ and μ denotes the minimal nonzero norm (or squared length), and is *dual-critical* if it achieves the global maximum γ'_n of $\{\mu(\Lambda)\,\mu(\Lambda^*)\}^{1/2}$ over all n-dimensional lattices. They prove that the absolutely extreme lattices in dimensions $n \leqslant 8$ are dual-extreme, and indeed are dual-critical for $n \leqslant 4$.

We call a lattice *isodual* if it is isometric, or geometrically congruent, to its dual, i.e., if it differs from its dual only by (possibly) a rotation and reflection. A lattice that is geometrically similar to its dual can always be rescaled to become isodual, when it will have determinant 1. While investigating lattices arising from Riemann surfaces, Buser and Sarnak [4] raised the question of finding the densest n-dimensional isodual lattices, or

0022-314X/94 $6.00

equivalently of determining μ_n, the maximal value of $\mu(\Lambda)$ over all n-dimensional isodual lattices Λ. Clearly

$$\mu_n \leqslant \gamma'_n \leqslant \gamma_n, \tag{1}$$

where γ_n is the Hermite constant.

The densest lattices known in dimensions 1, 2, 4, 8, 12, 16, 24 and 48 are isodual [5], and so in particular we know the values $\mu_1 = \gamma_1$, $\mu_2 = \gamma_2$, $\mu_4 = \gamma_4$, and $\mu_8 = \gamma_8$.

In the present paper we classify all three-dimensional isodual lattices (Theorem 1) and show that the densest is a certain lattice we call the m.c.c. lattice (Theorem 2), thus proving that $\mu_3 = 1/2 + \sqrt{1/2}$. This lattice, which also makes a brief appearance in [3], is described in Section 3. In Theorem 3 we show that it is also the most economical isodual covering.

We have also found reasonably dense isodual lattices in all dimensions up to 24. The best of these are shown in Table I. The table also gives lower bounds for γ'_n and γ_n. We expect that many of these bounds are in fact the exact values (although the lower bounds for μ_9, μ_{11}, μ_{13}, μ_{15} are probably weak).

TABLE I

Highest Minimal Norm μ_n of n-Dimensional Isodual Lattice, Corresponding Lattice Λ, Bergé–Martinet Constant γ'_n, Hermite Constant γ_n

n	μ_n	Λ	γ'_n	γ_n
1	1	I_1	1	1
2	$\sqrt{4/3} = 1.1547$	$\sqrt{1/3}A_2$	$\sqrt{4/3} = 1.1547$	1.1547
3	$1/2 + \sqrt{1/2} = 1.2071$	m.c.c.	$\sqrt{3/2} = 1.2247$	1.2599
4	$\sqrt{2} = 1.4142$	$\sqrt{1/2}D_4$	$\sqrt{2}$	1.4142
5	$\geqslant 7/5 = 1.4$	$(A_4 5_1)^+$	$\geqslant \sqrt{2}$	1.5157
6	$\geqslant 1 + \sqrt{1/3} = 1.5773$	$(D_4\, \sqrt{4/3}A_2)^+$	$\geqslant \sqrt{8/3} = 1.6329$	1.6653
7	$\geqslant 5/3 = 1.6666$	$(E_6 3_1)^+$	$\geqslant \sqrt{3} = 1.7320$	1.8114
8	2	E_8	2	2
9	$\geqslant 12/7 = 1.7142$	$(A_6 7_3)^+$	$\geqslant \sqrt{16/5} = 1.7888$	$\geqslant 2$
10	$\geqslant 2$	$D_{10}^+; Q_{10}$	$\geqslant 2$	$\geqslant 2.0583$
11	$\geqslant 7/4 = 1.75$	$(A_7 A_3 8_1)^+$	$\geqslant 2$	$\geqslant 2.1401$
12	$\geqslant \sqrt{16/3} = 2.3094$	K_{12}	$\geqslant \sqrt{16/3} = 2.3094$	$\geqslant 2.3094$
13	$\geqslant 2$	$(A_{11} A_2)^+$	$\geqslant \sqrt{24/5} = 2.1908$	$\geqslant 2.3563$
14	$\geqslant \sqrt{16/3} = 2.3094$	Q_{14}	$\geqslant \sqrt{16/3} = 2.3094$	$\geqslant 2.4886$
15	$\geqslant 2$	A_{15}^+	$\geqslant \sqrt{6} = 2.4494$	$\geqslant 2.6390$
16	$\geqslant \sqrt{8} = 2.8284$	Λ_{16}	$\geqslant \sqrt{8} = 2.8284$	$\geqslant 2.8284$
23	$\geqslant 3$	O_{23}	$\geqslant \sqrt{12} = 3.4641$	$\geqslant 3.7660$
24	$\geqslant 4$	Λ_{24}	$\geqslant 4$	$\geqslant 4$

The lattices are described using the notation of [5–7]. In particular, if L is a lattice with Gram matrix A, ${}^cL$ has Gram matrix cA. Also, $(LM)^+$ denotes a lattice obtained by gluing together component lattices L and M (cf. [5], Chap. 4).

The isodual lattices in dimensions 5–7, 9–11, 14 mentioned in the table are given in Section 6.

The lower bounds for the Hermite constant are obtained from the lattices given in Table 1.2 of [5]. The entries in the γ'_n column for $n \leqslant 9$ are taken from [3]. The entries in the γ'_n column for $n \geqslant 10$ are based on the same lattices used for the Hermite constant, except for dimensions 10 (when we use D_{10}^+) and 13 (when we use the lattice $Q_{13}(2)$ of [7]). Decimal expansions have been truncated to four places.

This paper makes use of the terminology for describing three-dimensional lattices introduced in [8].

2. The Classification of Three-Dimensional Isodual Lattices

Theorem 1. (i) *A decomposable three-dimensional lattice is isodual if and only if it is equivalent to one with Gram matrix*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & -h \\ 0 & -h & \beta \end{bmatrix} \tag{2}$$

where

$$\alpha\beta - h^2 = 1 \quad \text{and} \quad 0 \leqslant 2h \leqslant \alpha \leqslant \beta. \tag{3}$$

(ii) *An indecomposable three-dimensional lattice is isodual if and only if it is equivalent to one with Gram matrix*

$$\frac{1}{2-\alpha\beta}\begin{bmatrix} \dfrac{2\alpha}{\beta} & -\alpha\beta & -\alpha(2-\beta) \\ -\alpha\beta & \dfrac{2\beta}{\alpha} & \dfrac{2\beta(1-\alpha)}{\alpha} \\ -\alpha(2-\beta) & \dfrac{-2\beta(1-\alpha)}{\alpha} & \dfrac{\alpha^2\beta + 2\alpha + 2\beta - 4\alpha\beta}{\alpha} \end{bmatrix} \tag{4}$$

where $0 < \alpha \leqslant \beta < 1$.

Proof. (i) is easily established using the fact that every one- and two-dimensional lattice of determinant 1 is isodual. (ii) We recall the main result of [8], which is that each three-dimensional lattice is uniquely represented by a projective plane of order 2 labeled with seven numbers, the *conjugate norms* or *conorms* of the lattice, whose minimum is 0 and whose support is not contained in a proper subspace. Two lattices are isomorphic if and only if the corresponding labelings differ only by an automorphism of the plane. (The conorms are the familiar Selling parameters [12] supplemented by 0, but for several reasons—see [8]—it is preferable to work with a set of seven numbers rather than six.)

Let Λ be an indecomposable three-dimensional isodual lattice, with dual Λ^*. From Section of [8] the isoduality of Λ implies that just one of the conorms is zero. We denote the conorms (as in [8]) by 0, p_{01}, p_{02}, p_{03}, p_{12}, p_{13}, p_{23}, as indicated in Fig. 1a. We also set $p_{ji} = p_{ij}$ for $0 \leqslant i < j \leqslant 3$.

The conorms for Λ^* are shown in Fig. 1b (see Fig. 8 of [8]), where

$$pp = \min\{p_{01}p_{23}, p_{02}p_{13}, p_{03}p_{12}\}, \tag{5}$$

$$pp_{ijk} = p_{ij}p_{jk} + p_{jk}p_{ki} + p_{ki}p_{ij}. \tag{6}$$

The minimum in (5) is attained just once, and without loss of generality we assume $pp = p_{02}p_{13}$, so that $p_{02}p_{13} < p_{01}p_{23}$ and $p_{02}p_{13} < p_{03}p_{13}$.

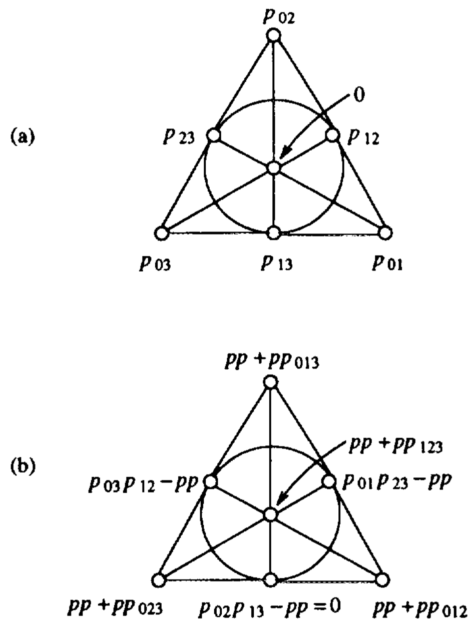

FIG. 1. (a) Projective plane labeled with conorms for Λ. (b) Conorms for dual lattice Λ^*.

The conorms in Figs. 1a and 1b must agree, up to an automorphism of the projective plane. This means that for some ordering π of $\{0, 1, 2, 3\}$ we have a set of six equations

$$pp_{012} + pp = p_{\pi_1\pi_3}, \qquad pp_{013} + pp = p_{\pi_0\pi_1},$$
$$pp_{023} + pp = p_{\pi_0\pi_3}, \qquad pp_{123} + pp = p_{\pi_2\pi_3},$$
$$p_{01}\, p_{23} - pp = p_{\pi_0\pi_3}, \qquad p_{03}\, p_{12} - pp = p_{\pi_1\pi_2},$$

to solve for $p_{01}, ..., p_{23}$. In 16 of the 24 cases, e.g., $\pi = \{0, 1, 2, 3\}$, there is no solution with all $p_{ij} > 0$. In four cases, e.g., $\pi = \{0, 3, 1, 2\}$, there is a two-parameter solution which after relabeling of the variables can be taken to be

$$p_{01} = \frac{\alpha(2-\beta)}{D}, \qquad p_{02} = \frac{\alpha\beta}{D},$$
$$p_{03} = \frac{2\alpha(1-\beta)}{\beta D}, \qquad p_{12} = \frac{2\beta(1-\alpha)}{\alpha D}, \tag{7}$$
$$p_{13} = \frac{2(1-\alpha)(1-\beta)}{D}, \qquad p_{23} = \frac{\beta(2-\alpha)}{D},$$

where $D = 2 - \alpha\beta$ and $0 < \alpha$, $\beta < 1$. The corresponding Gram matrix is (4). There is an obvious symmetry interchanging α and β, so we may assume $\alpha \leqslant \beta$. In the remaining four cases, e.g., $\pi = \{0, 1, 3, 2\}$, there is one solution, the m.c.c. lattice (see Section 3), which is in fact the case $\alpha = \beta = 2 - \sqrt{2}$ of the two-parameter family. ▮

Remark. The indecomposable isodual lattice in (ii) is generated by the three vectors

$$\left(-\frac{\lambda\alpha}{2}, \alpha, \frac{\alpha}{\lambda}\right), \quad \left(\frac{\lambda}{\alpha}, 0, 0\right), \quad \left(\lambda - \frac{\lambda}{\alpha}, -1, 0\right), \tag{8}$$

where

$$\lambda = \sqrt{2\alpha\beta/(2-\alpha\beta)}.$$

These vectors have Gram matrix (4).

3. The Mean Centered-Cuboidal Lattice

This is the case $\alpha = \beta = 2 - \sqrt{2}$, $\lambda = \sqrt{\sqrt{2} - 1}$ of (4), (8), but can be obtained more simply as follows. Consider the lattice generated by the vectors $(\pm u, \pm v, 0)$ and $(0, \pm v, \pm v)$ for real numbers u and v. If $\sigma := u/v = 1$ this is equivalent to the face-centered cubic lattice, if $\sigma = \sqrt{1/2}$ this is equivalent to the body-centered cubic lattice, and if $\sigma = \sqrt[4]{1/2}$ this is equivalent to what we call the mean centered-cuboidal (or m.c.c.) lattice.

When scaled so that it has determinant 1, the m.c.c. lattice has integral basis

$$(\sqrt{1/2}, \sqrt[4]{1/2}, 0), \qquad (\sqrt{1/2}, 0, \sqrt[4]{1/2}), \qquad (0, \sqrt[4]{1/2}, \sqrt[4]{1/2}), \tag{9}$$

Gram matrix

$$1/2 \begin{bmatrix} 1+\sqrt{2} & -1 & -1 \\ -1 & 1+\sqrt{2} & 1-\sqrt{2} \\ -1 & 1-\sqrt{2} & 1+\sqrt{2} \end{bmatrix}, \tag{10}$$

minimal norm $1/2 + \sqrt{1/2}$, center density $\delta = 0.1657...$ (between that of the face-centered and body-centered cubic lattices), covering radius $3^{0.5}2^{-1.25}$, kissing number 8, theta series

$$1 + 8q^{1/2+\sqrt{1/2}} + 4q^{\sqrt{2}} + 2q^2 + 4q^{2\sqrt{2}} + 8q^{2+\sqrt{2}} + 16q^{1/2+5/\sqrt{2}} + \cdots,$$

automorphism group of order 16, and is isodual.

Incidentally, if we take $\sigma = \sqrt{1/3}$ we obtain another interesting lattice, the "axial centered-cuboidal" lattice. After rescaling this has Gram matrix

$$\begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 1 \\ 2 & 1 & 4 \end{bmatrix},$$

determinant 36, minimal norm 4, center density $\delta = 1/6$, and kissing number 10. It can be obtained by stacking layers of equal spheres placed in the hexagonal lattice (or A_2) arrangement, with the spheres in each layer placed over some of the cols (points midway between two neighboring lattice points) of the layer beneath. Patterson [11] and Fields [9] have shown that this is the least dense lattice with kissing number 10.

4. The Densest Three-Dimensional Isodual Lattice

THEOREM 2. *The m.c.c. lattice is the unique densest three-dimensional isodual lattice.*

Proof. In [8] quantities called the *Voronoi norms* or *vonorms* associated with a lattice Λ are also introduced. These are the minimal nonzero norms of the classes of $\Lambda/2\Lambda$. The conorms mentioned in Section 2 are, apart from a scale factor, the discrete Fourier transform of the vonorms. The minimal nonzero vonorm is equal to the minimal nonzero norm $\mu(\Lambda)$ of the lattice.

(i) If Λ is decomposable and defined by (2), there are just four nonzero conorms, which we can take to be $p_{01}=\alpha-h$, $p_{02}=\beta-h$, $p_{03}=1$, $p_{12}=h$. Therefore $\mu(\Lambda)\leqslant 1$.

(ii) Suppose Λ is indecomposable, defined by (4), and has the conorms given in (7). The corresponding vonorms $p_{0|123}$, $p_{1|023}$, $p_{2|013}$, and $p_{3|012}$ are respectively equal to

$$\frac{2\alpha}{\beta D}, \quad \frac{\alpha^2\beta+2\alpha+2\beta-4\alpha\beta}{\alpha D}, \quad \frac{2\beta}{\alpha D}, \quad \frac{\alpha\beta^2+2\alpha+2\beta-4\alpha\beta}{\beta D}. \tag{11}$$

For the m.c.c. lattice, when $\alpha=\beta=2-\sqrt{2}$, all four quantities are equal to $1/2+\sqrt{1/2}$, the other three vonorms are $\sqrt{2}$, and $\mu=1.2071\ldots$.

It is now an elementary calculation to verify that the sum of the four quantities (11) is maximized over the region $0<\alpha\leqslant\beta<1$ at the point $\alpha=\beta=2-\sqrt{2}$. At all other points it is strictly smaller. Therefore for any other lattice some vonorm is strictly less than $1/2+\sqrt{1/2}$, and the density is less than that of the c.c.c. lattice. ∎

5. The Most Efficient Three-Dimensional Isodual Covering Lattice

THEOREM 3. *The m.c.c. lattice is the unique most efficient three-dimensional isodual lattice covering.*

Proof. Let Λ be a three-dimensional lattice with conorms 0, $p_{01}, \ldots, p_{23}$ (where we allow some of the p_{ij} to be zero). It is a consequence of Eqs. (11) and (12) of [2] that the squared covering radius R of Λ is given by

$$R=\frac{1}{4\det\Lambda}(S_1\det\Lambda-S_2-4S_3), \tag{12}$$

where

$$S_1=p_{01}+p_{02}+p_{03}+p_{12}+p_{13}+p_{23},$$

$$S_2=\sum p_W p_X p_Y p_Z,$$

the sum being taken over all products of four conorms in Fig. 1a whose support is the complement of a triangle, and

$$S_3 = \min\{p_{01}p_{12}p_{23}p_{30}, p_{02}p_{21}p_{13}p_{30}, p_{01}p_{13}p_{32}p_{20}\}. \qquad (13)$$

For the m.c.c. lattice, $R = 3\sqrt{2}/8 = 0.5303\ldots$. (i) For a decomposable lattice defined by (2) we find

$$R = \tfrac{1}{4}\{\alpha + \beta + 1 - h - h(\alpha - h)(\beta - h)\},$$

which has a minimum of $2\sqrt{3}/9 + 1/4 = 0.6349\ldots$ in the region (3). (ii) Let Λ be an indecomposable lattice defined by (4) and with conorms (7). The minimum in (13) is attained (I) at $p_{01}p_{13}p_{32}p_{20}$ if $\alpha \leqslant 2(1-\beta)/(2-\beta)$, or (II) at $p_{02}p_{21}p_{13}p_{30}$ if $\alpha \geqslant 2(1-\beta)/(2-\beta)$. In each case we find that R has no local minimum in the interior of the region, and that on the boundary there is a unique local minimum at $\alpha = \beta = 2 - \sqrt{2}$. ∎

6. Higher-Dimensional Isodual Lattices

Dimensions 5, 6, 7. Our constructions of isodual lattices in dimensions 3 (see Section 3), 5, 6, and 7 (see Table I) can be described in a uniform manner. In each dimension, we find a path in lattice space leading from the absolutely extreme lattice Λ to its dual Λ^*, coordinatized by a parameter σ, with $\sigma = s$ (resp. $\sigma = s^*$) corresponding to Λ (resp. Λ^*). Then the isodual lattice $m\Lambda$ has $\sigma = \sqrt{ss^*}$.

In dimension 6, using the terminology of Chapter 4 of [5], we construct a one-parameter family of lattices by gluing the root lattice D_4 to a scaled copy ${}^cA_2$ of the root lattice A_2. We denote the glue vectors of D_4 by [1], [2], [3], and we let v_1, v_2, v_3 be three minimal vectors of ${}^cA_2$ that add to zero. Then $M(c)$, $c > 0$, is generated by $D_4 \oplus {}^cA_2$ and the vectors $([i]; v_i/2)$, $1 \leqslant i \leqslant 3$. It is easy to verify that $M(2)$ is equivalent to the root lattice E_6, $M(2/3)$ to E_6^*, and that $M(\sqrt{4/3})$ is an isodual lattice mE_6 with determinant 1, minimal norm $1 + 1/\sqrt{3}$, kissing number 48, center density $\delta = 0.06132\ldots$ (only slightly worse than the densities $0.07216\ldots$ of E_6 and $0.06415\ldots$ of E_6^*), and automorphism group of order $2(2^3 4!)\, 3! = 2304$.

There are similar constructions in dimensions 5 and 7. Let $(5c)_1$ denote the one-dimensional lattice of determinant $5c$ generated by a vector v of norm $5c$. We define $L(c)$ to be the lattice generated by $A_4 \oplus (5c)_1$ and ([1]; $2v/5$), where $[1] = 4/5, -1/5, -1/5, -1/5, -1/5$ is the first glue vector for A_4. Then $L(4)$ is equivalent to D_5, $L(1/4)$ to D_5^*, and $L(1)$ is an isodual lattice mD_5 with determinant 1, minimal norm 7/5, kissing number 20, and center density $\delta = 0.07247\ldots$. Similarly, in dimension 7 we glue E_6 to 3_1.

Dimensions 5, 7, 9, 11. The isodual lattices in dimensions 5, 7 (again), 9, and 11 in Table I also have a uniform description: for $n = 1, 2, 3, 4$ we glue the sublattice of the simple cubic lattice I_{n+1} perpendicular to the vector $2\,1^n$ to the sublattice of I_{n+4} perpendicular to the vector 1^{n+4} (the latter is A_{n+3}), obtaining an isodual lattice in dimension $2n+3$.

Dimensions 10 *and* 14. Several 10-dimensional lattices have $\mu(\Lambda) = 2$: for example, the lattice D_{10}^{+} [5, Chap. 4], or a scaled form of the lattice Q_{10} associated with the "duads" and "synthemes" of the symmetric group $\mathbf{S}_6$. This lattice is most simply obtained by applying Construction A [5] to the [10, 5, 4] binary code with generator matrix

$$\begin{bmatrix} 1 & & & & & 1 & 1 & 1 & 0 & 0 \\ & 1 & & & & 1 & 1 & 0 & 1 & 0 \\ & & 1 & & & 1 & 0 & 1 & 0 & 1 \\ & & & 1 & & 0 & 1 & 0 & 1 & 1 \\ & & & & 1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

The dual of this code is a permutation of itself [13], [10]. In this form, Q_{10} has determinant 2^{10}, minimal norm 4, kissing number 260, and automorphism group $2^{10} \cdot \mathbf{S}_6$. This lattice may also be obtained by gluing ${}^{1/2}Q_9(5)$ to 5_1, where $Q_9(5)$ was defined in [7].

In dimension 14 our isodual lattice is a scaled form of the lattice Q_{14} associated with the simple group $\mathbf{G}_2(3)$ [1, p. 60]. The lattice Q_{14} has determinant 3^7, minimal norm 4, kissing number 756, and automorphism group $\mathbf{G}_2(3) \cdot 2$. It may also be obtained by gluing a scaled copy of the lattice $Q_{13}(4)$ of [7] to 4_1.

References

1. J. H. Conway, R. T. Curtis, S. P. Norton, R. A. Parker, and R. A. Wilson, "ATLAS of Finite Groups," Oxford Univ. Press, London/New York, 1985.
2. E. S. Barnes and N. J. A. Sloane, The optimal lattice quantizer in three dimensions, *SIAM J. Alg. Discrete Methods* **4** (1983), 30–41.
3. A.-M. Bergé and J. Martinet, Sur un problème de dualité lié aux sphères en géometrie des nombres, *J. Number Theory* **32** (1989), 14–42.
4. P. Buser and P. Sarnak, On the period matrix of a Riemann surface of large genus, (with an Appendix by J. H. Conway and N. J. A. Sloane), *Invent. Math.* **117** (1994), 27–56.
5. J. H. Conway and N. J. A. Sloane, "Sphere-Packings, Lattices and Groups," Springer-Verlag, New York, 2nd ed., 1992.
6. J. H. Conway and N. J. A. Sloane, Low-dimensional lattices. I. Quadratic forms of small determinant, *Proc. Roy. Soc. London Ser. A* **418** (1988), 17–41.

7. J. H. CONWAY AND N. J. A. SLOANE, Low-dimensional lattices. II. Subgroups of $GL(n, \mathbb{Z})$, *Proc. Roy. Soc. London Ser. A* **419** (1988), 29–68.
8. J. H. CONWAY AND N. J. A. SLOANE, Low-dimensional lattices. VI. Voronoi reduction of three-dimensional lattices, *Proc. Roy. Soc. London Ser. A* **436** (1992), 55–68.
9. K. L. FIELDS, The fragile lattice packings of spheres in three-dimensional space, *Acta Cryst. Sect. A* **36** (1980), 194–197.
10. G. T. KENNEDY AND V. PLESS, On designs and formally self-dual codes, preprint.
11. A. PATTERSON, Crystal lattice models based on the close packing of spheres, *Rev. Sci. Instrum.* **12** (1941), 206–211.
12. E. SELLING, Ueber die binären und ternaren quadratischen Formen, *J. Reine Angew. Math.* **77** (1874), 143–229.
13. H. N. WARD, A restriction on the weight enumerator of a self-dual code, *J. Comb. Theory Ser. A* **21** (1976), 253–255.