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The Persistence of a Number

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Problem: Find the next term in the following sequence:

$$679 \rightarrow 378 \rightarrow 168 \rightarrow 48 \rightarrow 32 \rightarrow ?$$

Answer: 6. The reason is that each term is the product of the digits of the preceding term. No matter what positive integer A_1 we start with, by successively multiplying the digits together we eventually terminate with 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. The number of steps for this to happen is called the *persistence* of A_1. Thus the persistence of 679 is 5. In fact 679 is the smallest number with persistence 5.

The following table gives the smallest numbers with persistence less than or equal to 11:

Persistence	*Number*
1	10
2	25
3	39
4	77
5	679
6	6788
7	68889
8	2677889
9	26888999
10	3778888999
11	277777788888899

For example, the last number gives the sequence:

$$277777788888899 \rightarrow 4996238671872 \rightarrow 438939648 \rightarrow 4478976 \rightarrow 338688 \rightarrow 27648 \rightarrow 2688 \rightarrow 768 \rightarrow 336 \rightarrow 54 \rightarrow 20 \rightarrow 0.$$

No number less than 10^{50} has persistence greater than 11, and it is conjectured that there is a number c such that no number has persistence greater than c.

The table was found by computer, using the fact that the second number A_2 in the sequence $A_1 \to A_2 \to \ldots$ must be of the form $2^i 3^j 7^k$ or $3^i 5^j 7^k$. (Since A_2 is the product of the digits of A_1, it cannot contain a prime greater than 7, and if the persistence of A_1 is greater than or equal to 4, it cannot contain both a 2 and a 5.)

It is interesting to consider this problem in other bases. In base 2 the maximum persistence is 1. In base 3, the second term A_2 is 0 or a power of 2. Calculations suggest the conjecture that all powers of 2 above 2^{15} contain a zero when written in base 3. This is true up to 2^{500}, but a general proof seems difficult. If the conjecture is true it implies that the maximum possible persistence in base 3 is 3, as illustrated by the sequence:

$$222222222222222 \to 2^{15} = 1122221122 \to 1012 \to 0.$$

Therefore the final conjecture is that there is a number $c(b)$ such that the persistence in base b cannot exceed $c(b)$.