Asymptotic Performance of Multiple Description Lattice Quantizers

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Abstract — **The high-rate squared-error distortions of a balanced multiple description lattice vector quantizer are analyzed for a memoryless source with probability density function p, differential entropy $h(p) < \infty$, and lattice codebook Λ. For any $a \in (0,1)$ and rate pair (R,R), it is shown that the two-channel distortion $\bar{d}_0$ and the channel 1 (or channel 2) distortion $\bar{d}_s$ satisfy**

$$\lim_{R\to\infty} \bar{d}_0 2^{2R(1+a)} = G(\Lambda)2^{2h(p)}/4$$

and

$$\lim_{R\to\infty} \bar{d}_s 2^{2R(1-a)} = G(S_L)2^{2h(p)},$$

where $G(\Lambda)$ is the normalized second moment of a Voronoi cell of the lattice Λ and $G(S_L)$ is the normalized second moment of a sphere in L dimensions.

I. Introduction

We consider a two-channel multiple description quantization system for a discrete-memoryless source with differential entropy $h(p)$. The quantizer transmits information on each channel at rate R bits/sample. The mean-squared error when both channels work is denoted by $\bar{d}_0$ and when either channel works is denoted by $\bar{d}_s$.

It has been shown [1] that for a uniform entropy-coded multiple description quantizer and any $a \in (0,1)$ the distortions satisfy

$$\begin{aligned} \lim_{R\to\infty} \bar{d}_0(R)2^{2R(1+a)} &= \frac{1}{4}\left(\frac{2^{2h(p)}}{12}\right), \\ \lim_{R\to\infty} \bar{d}_s(R)2^{2R(1-a)} &= \left(\frac{2^{2h(p)}}{12}\right). \end{aligned} \quad (1)$$

On the other hand, by using a random quantizer argument it was shown [2] that by encoding vectors of infinite block length, it is possible to achieve distortions

$$\begin{aligned} \lim_{R\to\infty} \bar{d}_0(R)2^{2R(1+a)} &= \frac{1}{4}\left(\frac{2^{2h(p)}}{2\pi e}\right), \\ \lim_{R\to\infty} \bar{d}_s(R)2^{2R(1-a)} &= \left(\frac{2^{2h(p)}}{2\pi e}\right). \end{aligned} \quad (2)$$

Thus in multiple description quantization it is possible to achieve a reduction in the granular distortion by 1.53 dB, simultaneously for the two-channel and the side distortion.

The goal of this paper is to analyze constructions given in [3] for closing this "1.53 dB" gap. The system to be analyzed is illustrated in Fig. 1. Our approach is as follows. From classical quantization theory, we know that the gap between scalar quantization and the rate distortion bound may be closed by using vector quantizers with lattice codebooks. Certainly, by following this approach we can also close the gap between the two-channel distortion and the rate-distortion bound. In particular, this will allow us to replace the factor (1/12) in the expression for $\bar{d}_0$ in (1) with $G(\Lambda)$, the normalized second moment of the Voronoi region of a lattice point. The main question we address here is that of simultaneously reducing $\bar{d}_1$. How can such a reduction be achieved and what is the quantity that will replace the factor (1/12) in the expression for $\bar{d}_1$ in (1)? We will show through a constructive procedure that the distortion $\bar{d}_1$ can be reduced by solving a specific labeling problem. To our surprise, the quantity that replaces (1/12) is $G(S_L)$, the normalized second moment of a *sphere* in L dimensions.

For details the reader is referred to the full paper [4], which will be published elsewhere.

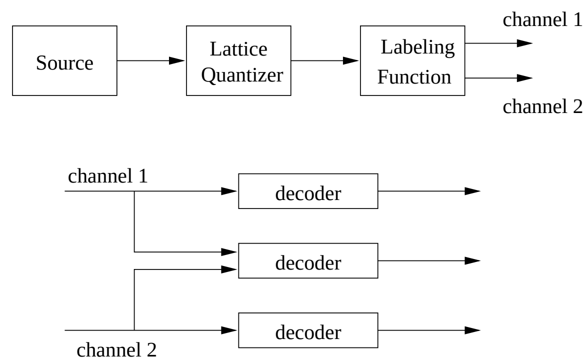

Figure 1: A multiple description vector quantizer with a lattice codebook.

References

[1] V. Vaishampayan and J.-C. Batllo, "Asymptotic analysis of multiple description quantizers," *IEEE Trans. Inform. Th.*, vol. 44, pp. 278–284, Jan. 1998.

[2] V. Vaishampayan, J.-C. Batllo and A. R. Calderbank, "On reducing granular distortion in multiple description quantization," in *Abstracts of Papers, IEEE Int. Symp. Inform. Theory*, Cambridge, MA, August 1998.

[3] S. D. Servetto, V. A. Vaishampayan and N. J. A. Sloane, "Multiple Description Lattice Vector Quantization", in *Proceedings 1999 Data Compression Conference*, pp. 13–22, IEEE Press, 1999.

[4] V. Vaishampayan, N. J. A. Sloane and Sergio D. Servetto, "Multiple description vector quantization with lattice codebooks: design and analysis," (submitted).